Facial strong parity colourings of plane graphs

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Parity edge-colourings

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Theorem (Pyber 1991)

The edges of any simple graph can be coloured with at most four colours in such a way that each colour class induces an odd subgraph.

Parity edge-colourings

Definition

Consider an edge-colouring of a plane graph G. We say that a colour c appears on a face α of G an odd number of times provided that an odd number of edges incident with α is coloured with colour c.

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Theorem (Pyber, 1991)

Let G be a simple 3-connected plane graph. Then the edges of G can be coloured with four colours $1, 2, 3, 4$ in such a way that for every colour $c \in \{1, 2, 3, 4\}$ and every face α of G no edge or an odd number of edges incident with α is coloured with c.

Strong parity chromatic index

Definition

Let G be a plane graph. The *facial strong parity chromatic index* $\chi'_p(G)$ of G is the minimum integer k of colours used in a proper edge colouring of G having property that for every colour $c \in \{1, 2, ..., k\}$ and every face α of G no edge or an odd number of edges incident with α is coloured with c.

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Theorem

Let G be a 3-connected plane graph, G^* be the dual of G and let $\chi'(G^*)$ be a chromatic index of G^*. Then

$$\chi'_p(G) \leq \chi'(G^*).$$

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$$\chi'_p(G) = 3.$$

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Theorem

Let C_n be a cycle on n vertices , then

$$\chi'_p(C_n) \leq 5.$$

Strong parity chromatic index

Two faces α and β of a plane graph G *touch* each other if they share a vertex in common. Two faces α and β of a plane graph G *influence* each other if there is a face that touches both α and β.

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Let G be a simple 2-connected plane graph in which no two faces of degrees ≥ 4 touch each other. Then

$$\chi'_p(G) \leq 6.$$

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Theorem

Let G be a simple 3-connected graph and let $m \geq 7$ be an integer. If no two faces of G of degree $\geq m+1$ influence each other, then

$$\chi'_p(G) \leq m+3.$$

Strong parity chromatic index

Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a bridgeless plane multigraph, then

$$\chi'_p(G) \leq 332.$$

Parity vertex-colourings

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Theorem (Czap, Jendroľ, 2008)

Let G be a simple 2-connected plane graph. Then the vertices of G can be coloured with four colours $1, 2, 3$, and 4 in such a way that on every face α of G at least one colour $c \in \{1, 2, 3, 4\}$ appears an odd number of times.

Strong parity chromatic number

Definition

Let G be a plane graph. The *facial strong parity chromatic number* $\chi_p(G)$ of G is the minimum integer k of colours used in a proper vertex colouring of G having property that for every colour $c \in \{1, 2, ..., k\}$ and every face α of G no vertex or an odd number of vertices incident with α is coloured with c.

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Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a 3-connected plane graph in which no two faces of degree ≥ 4 touch each other. Then

$$\chi_p(G) \leq 6.$$

Moreover the bound 6 is the best possible.

Strong parity chromatic number

Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a simple 3-connected plane graph such that no two faces of degree at least 5 influence each other. Then

$$\chi_p(G) \leq 8.$$

Strong parity chromatic number

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Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a simple 3-connected plane graph such that no two faces of degree at least 6 influence each other. Then

$$\chi_p(G) \leq 10.$$

Strong parity chromatic number

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Let G be a plane graph. A face α of degree i of G is called to be isolated if there is no face β of degree at least i touching α.

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Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a 2-connected plane graph in which every face of degree ≥ 4 is isolated. Then

$$\chi_p(G) \leq 12.$$

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Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a 3-connected plane graph in which any face of degree ≥ 5 is isolated. Then

$$\chi_p(G) \leq 18.$$

Strong parity chromatic index

Theorem (Czap, Jendroľ, Kardoš, 2009)

Let G be a 3-connected plane graph in which any face of degree ≥ 6 is isolated. Then

$$\chi_p(G) \leq 28.$$

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Conjecture 1 (Czap, Jendroľ, 2008)

There exists an absolute constant K such that for every simple 2-connected planar graph

$$\chi_p(G) \leq K.$$

Thanks for your attention