Regular covers of the uniform tessellations of the plane

Daniel Pellicer

Gordon Williams

Abstract polyhedra

2-cell embedding of a (possibly infinite) connected graph on a surface

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Forbidden

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- loops

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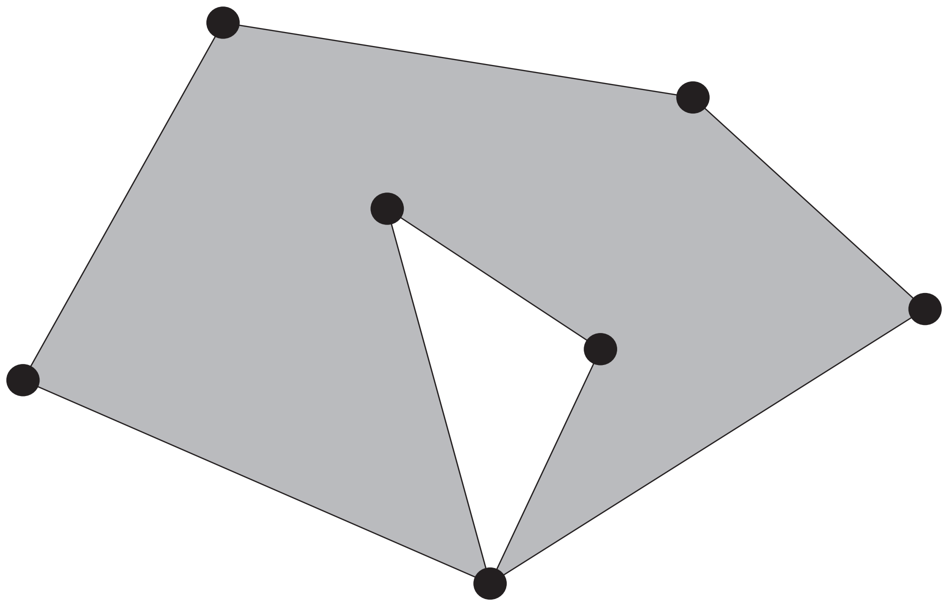
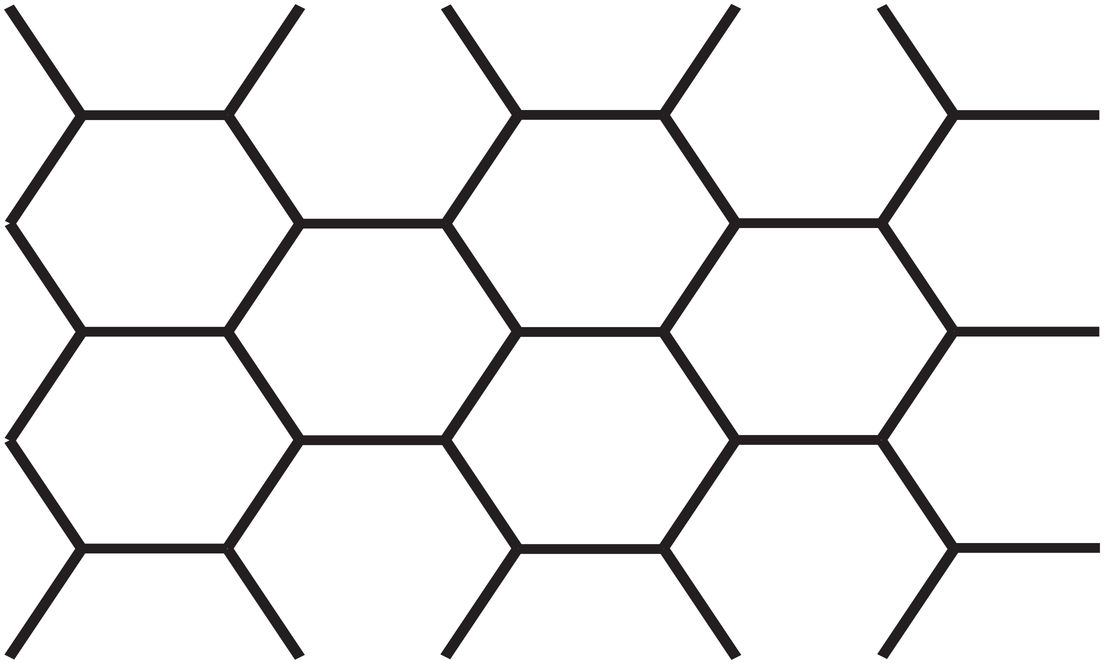
- 

Abstract polyhedra



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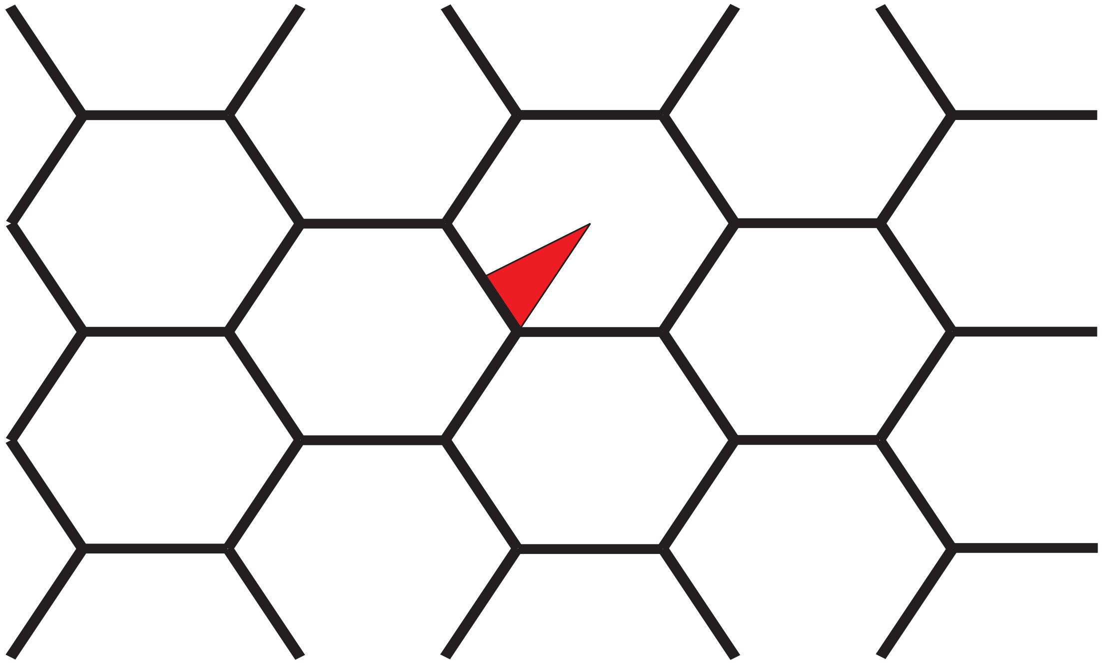

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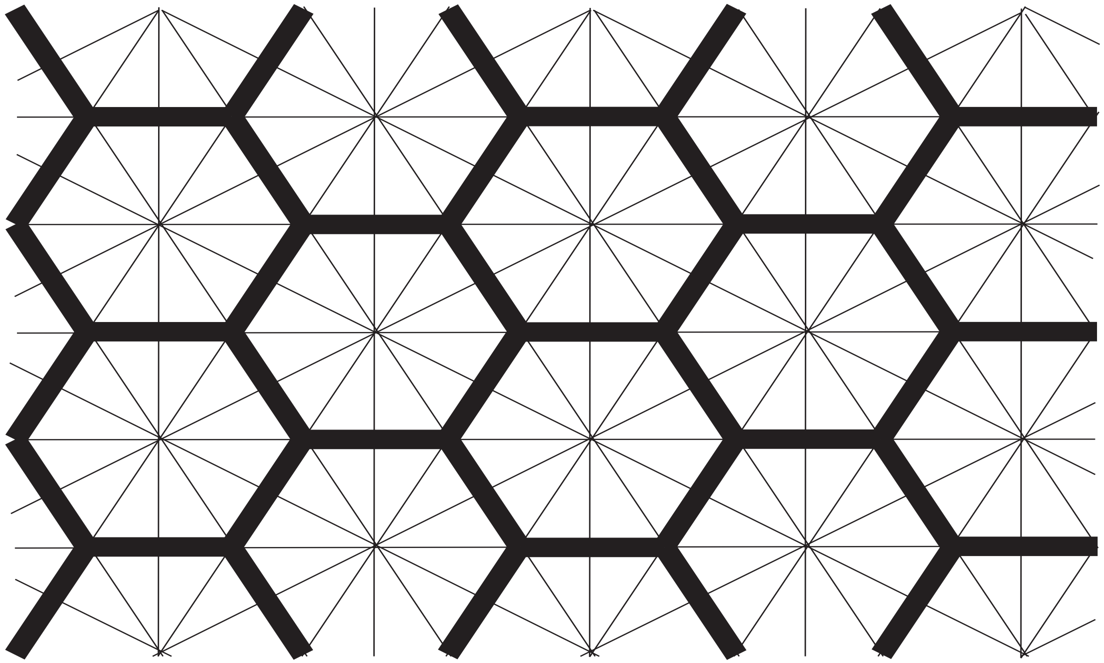

Abstract polyhedra

Regular $\longrightarrow$ automorphism group transitive on flags

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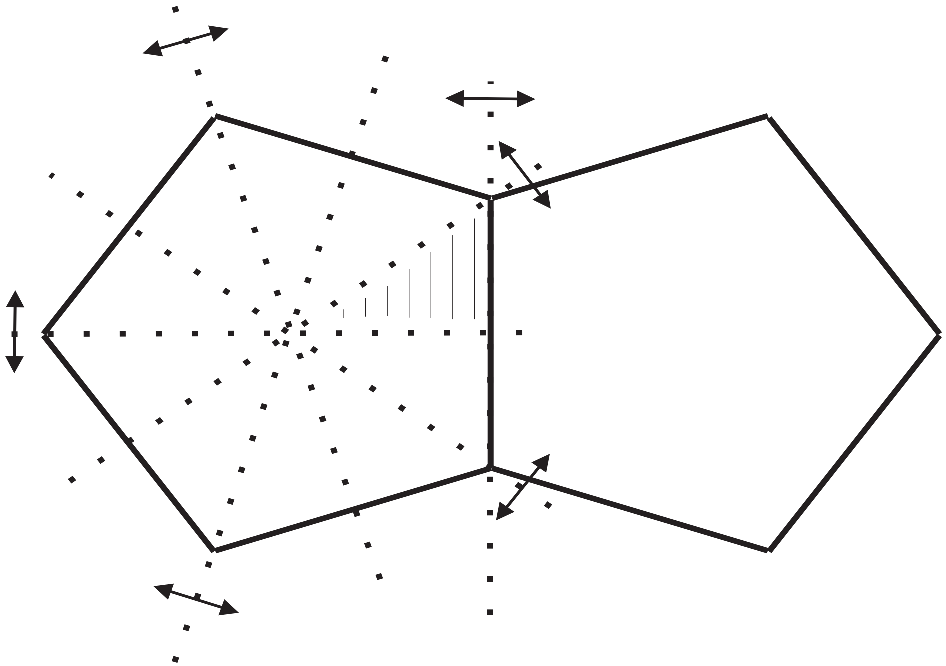
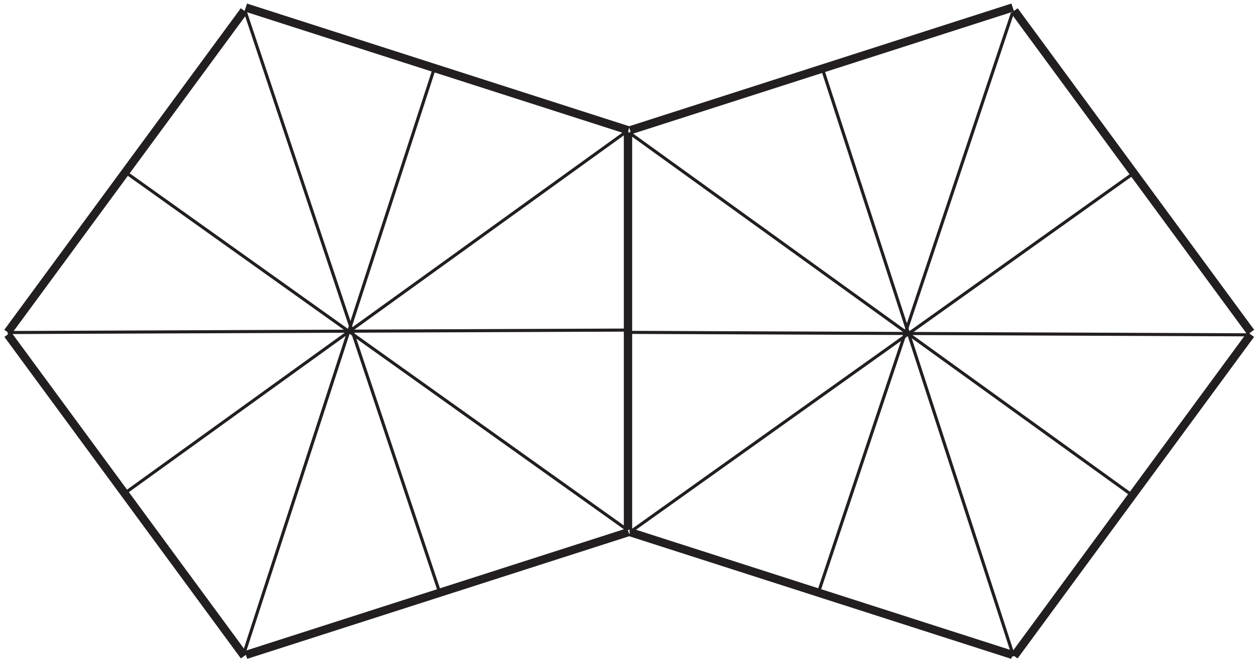


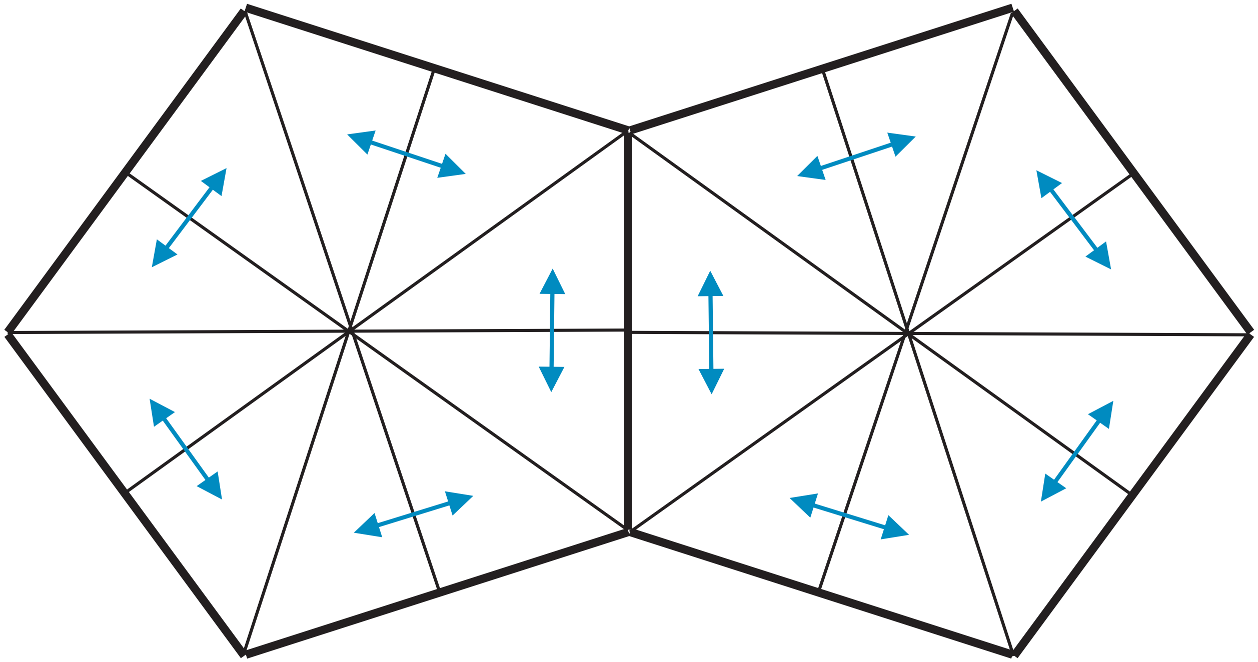
Monodromy group

Permutations of flags



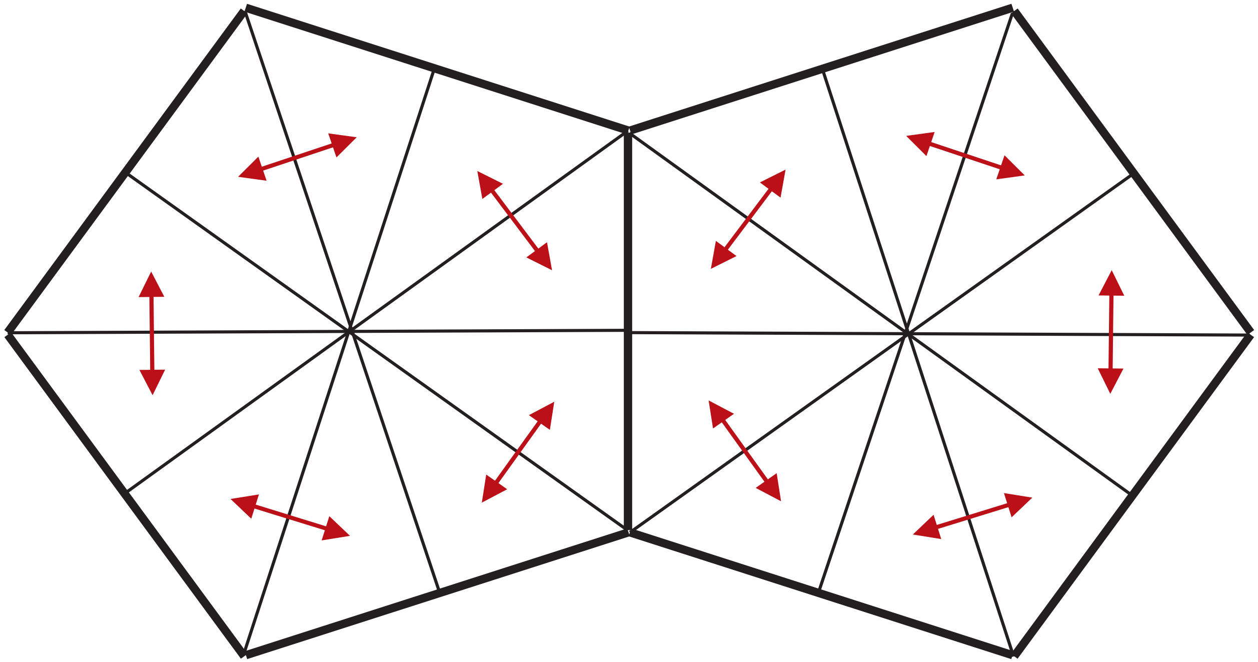
Monodromy group

r_0



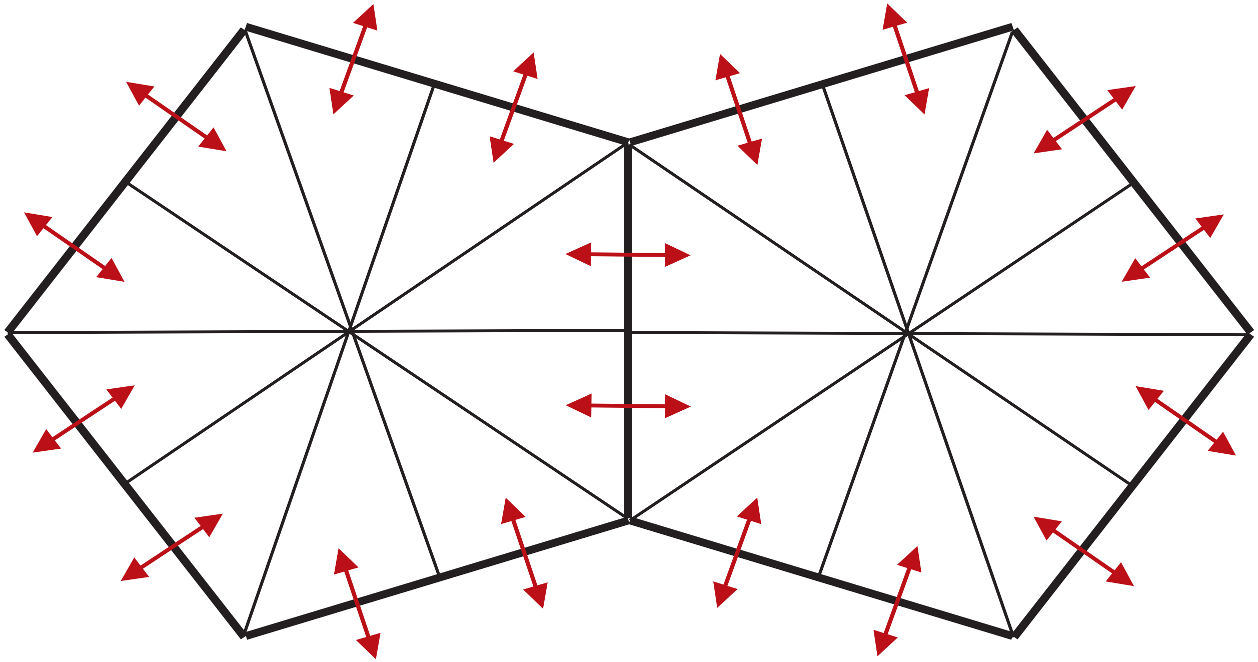
Monodromy group

r_1



Monodromy group

r_2



Monodromy group

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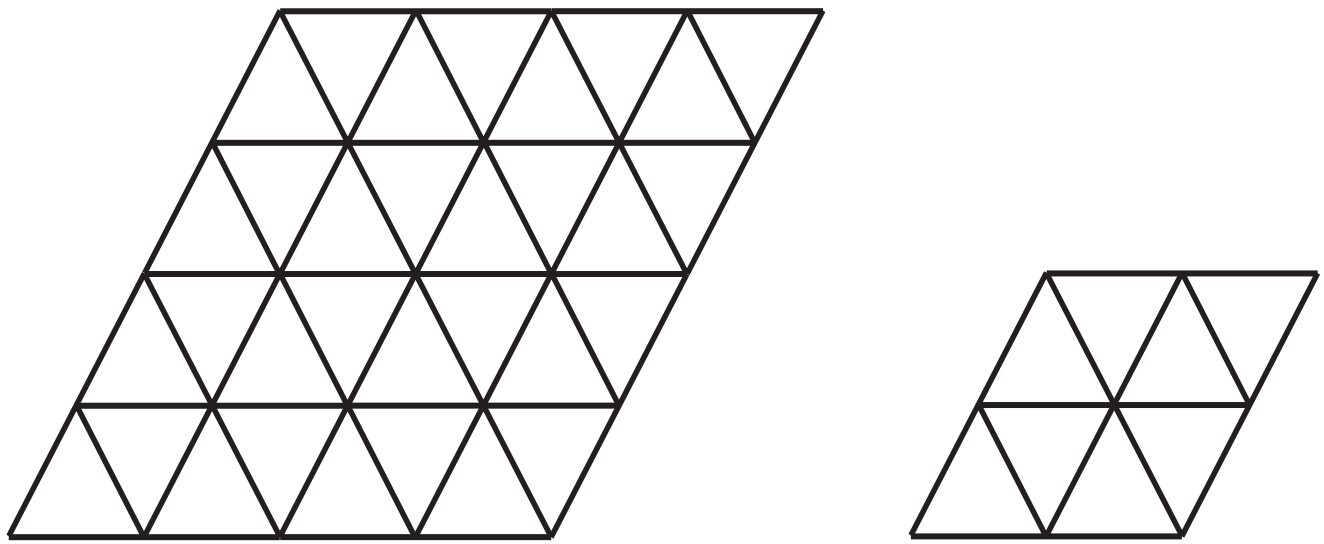
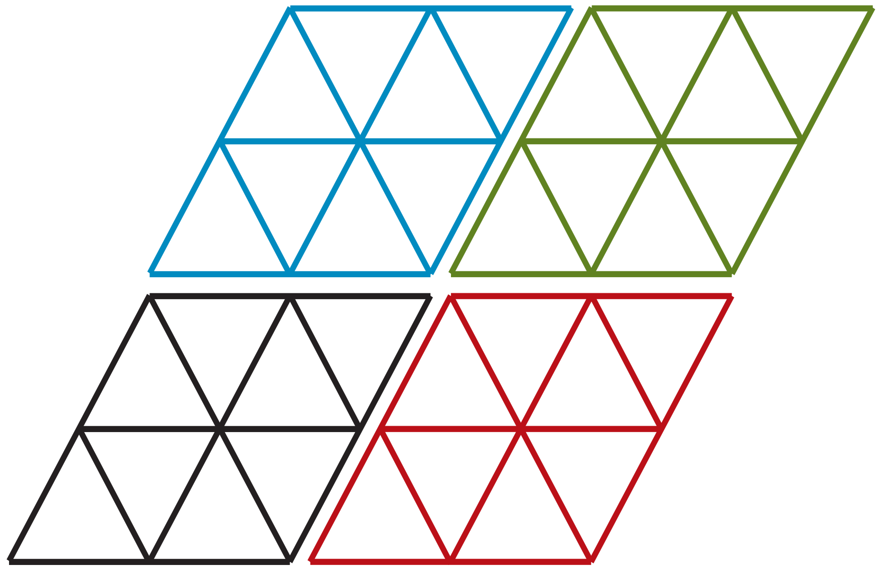
Regular $\mathcal{P}$ covers $\mathcal{Q}$ if there is an epimorphism

$$\begin{array}{ccc} \Gamma(\mathcal{P}) & \rightarrow & Mon(\mathcal{Q}) \\ \rho_i & \mapsto & r_i \end{array}$$

Covers



Covers



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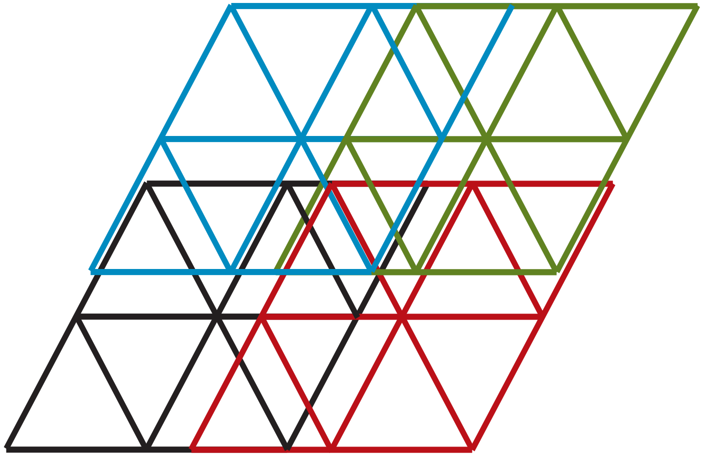

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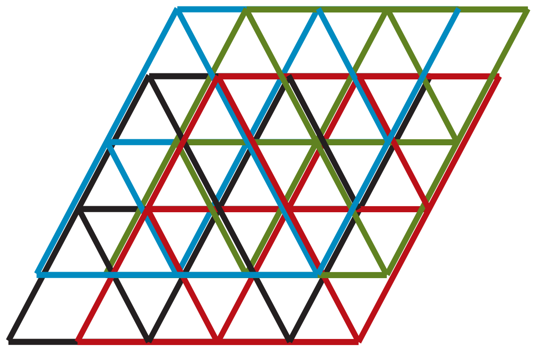

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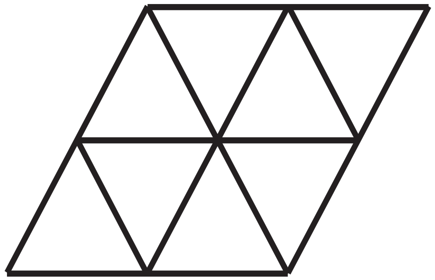

Covers

Every polyhedron has a (minimal) regular cover

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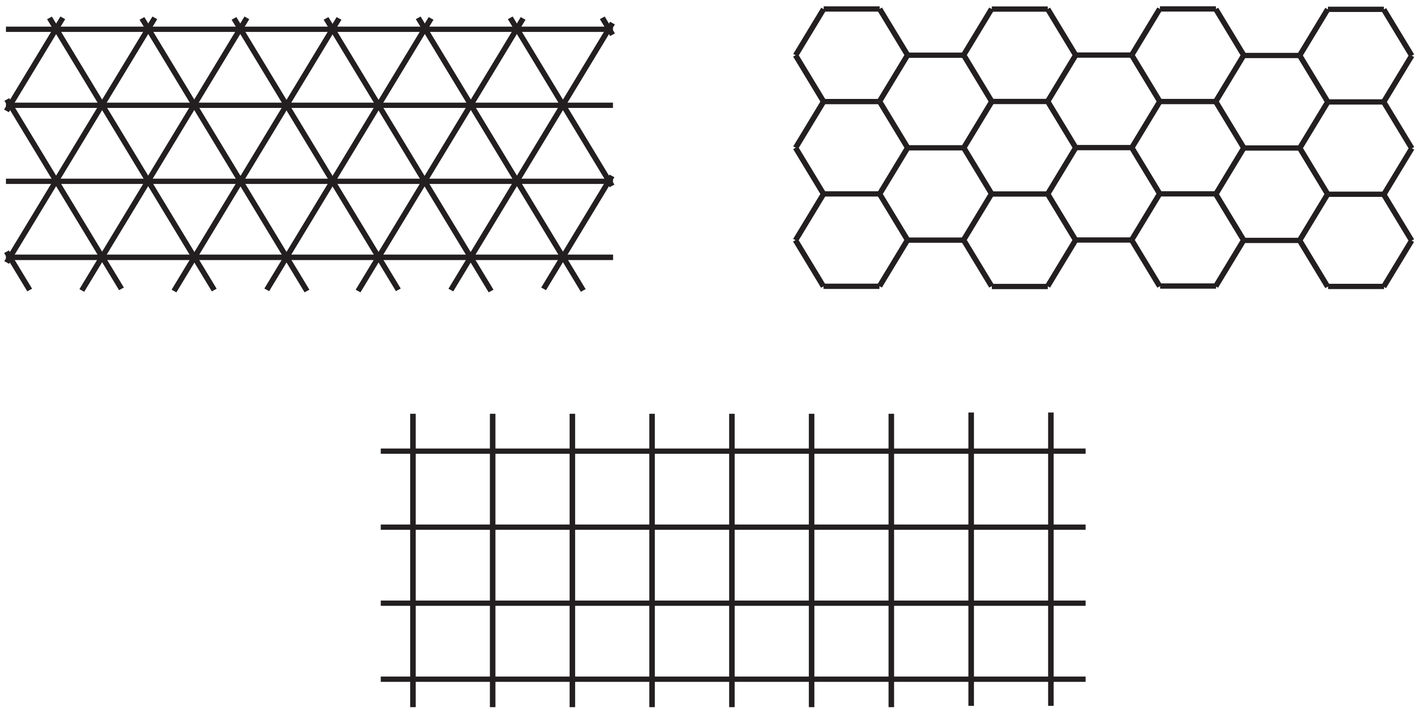
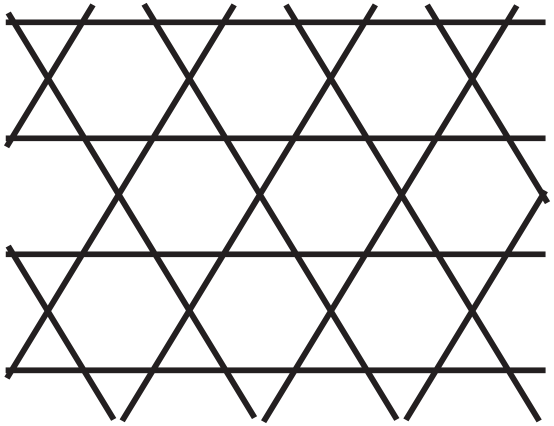
M. Hartley, G. Williams worked on the regular covers of the Archimedean solids

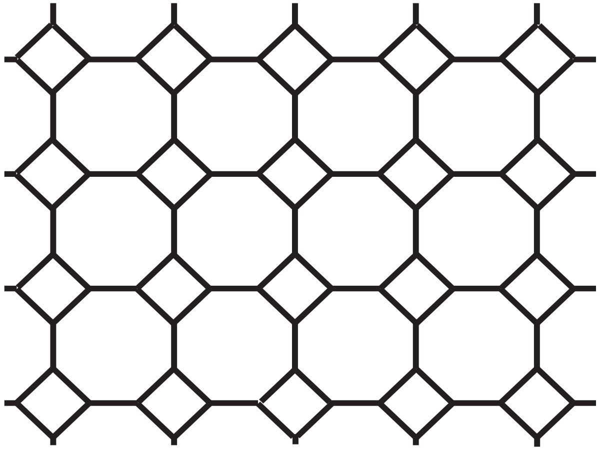
Regular tessellations



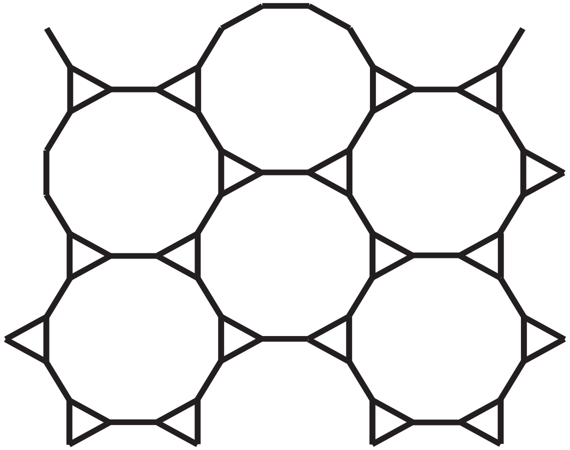
Uniform tessellations



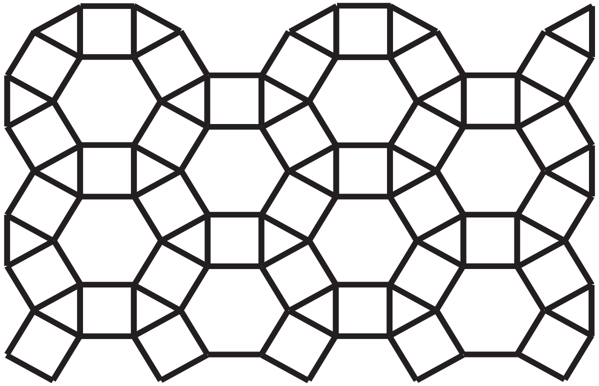
3.6.3.6



4.8.8

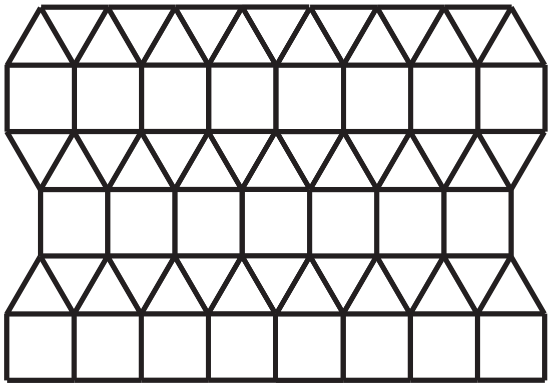


3.12.12

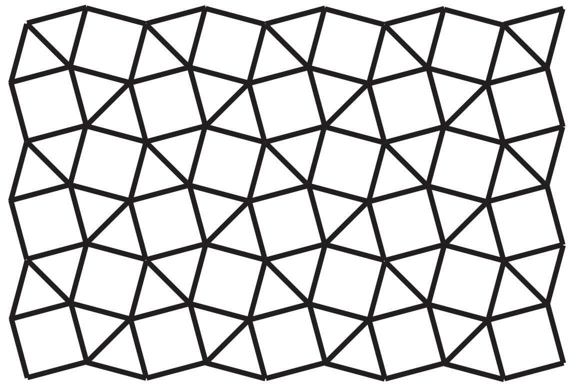


3.4.6.4

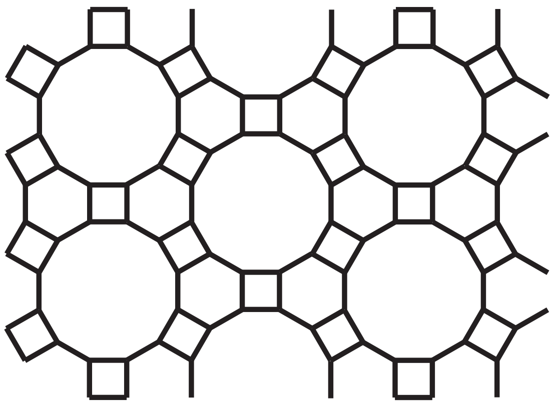
Uniform tessellations



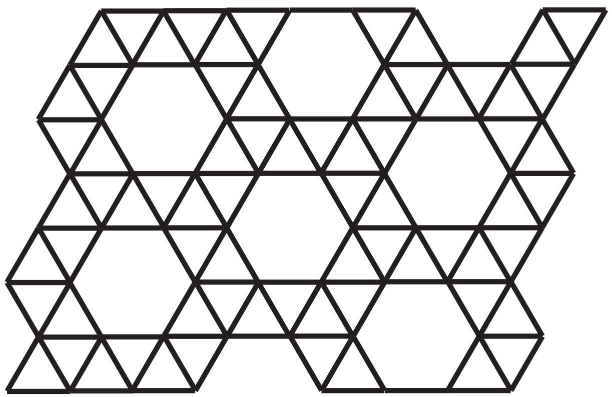
3.3.3.4.4



3.3.4.3.4

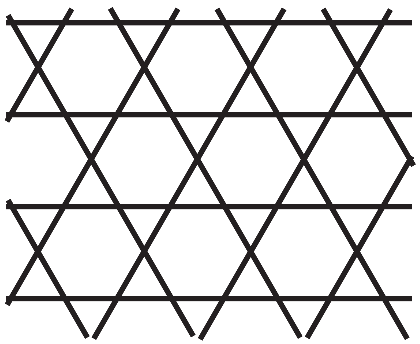


4.6.12

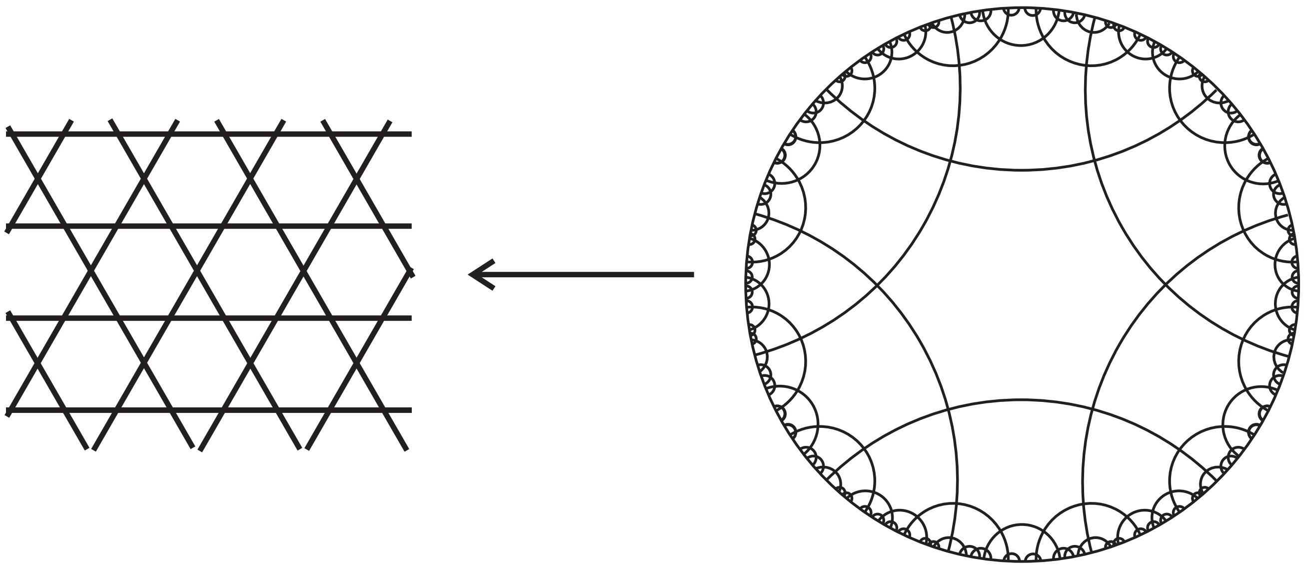


3.3.3.3.6

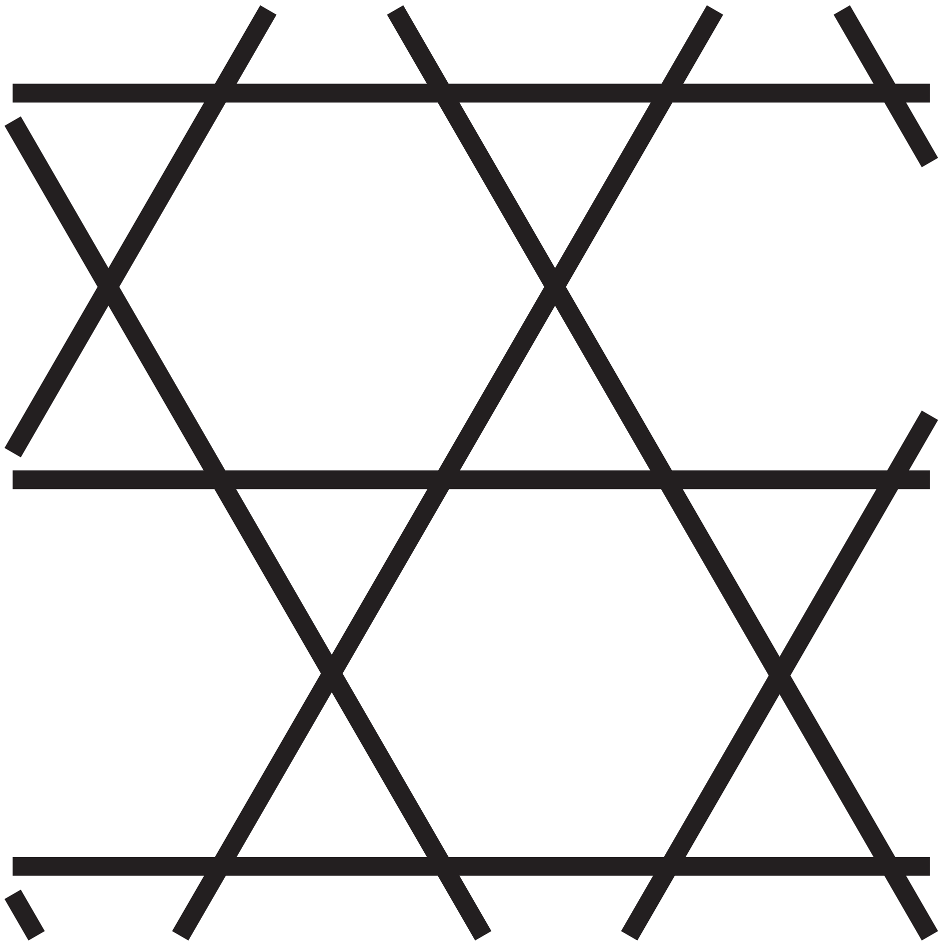
Universal cover



Universal cover



Stabilizer of a flag



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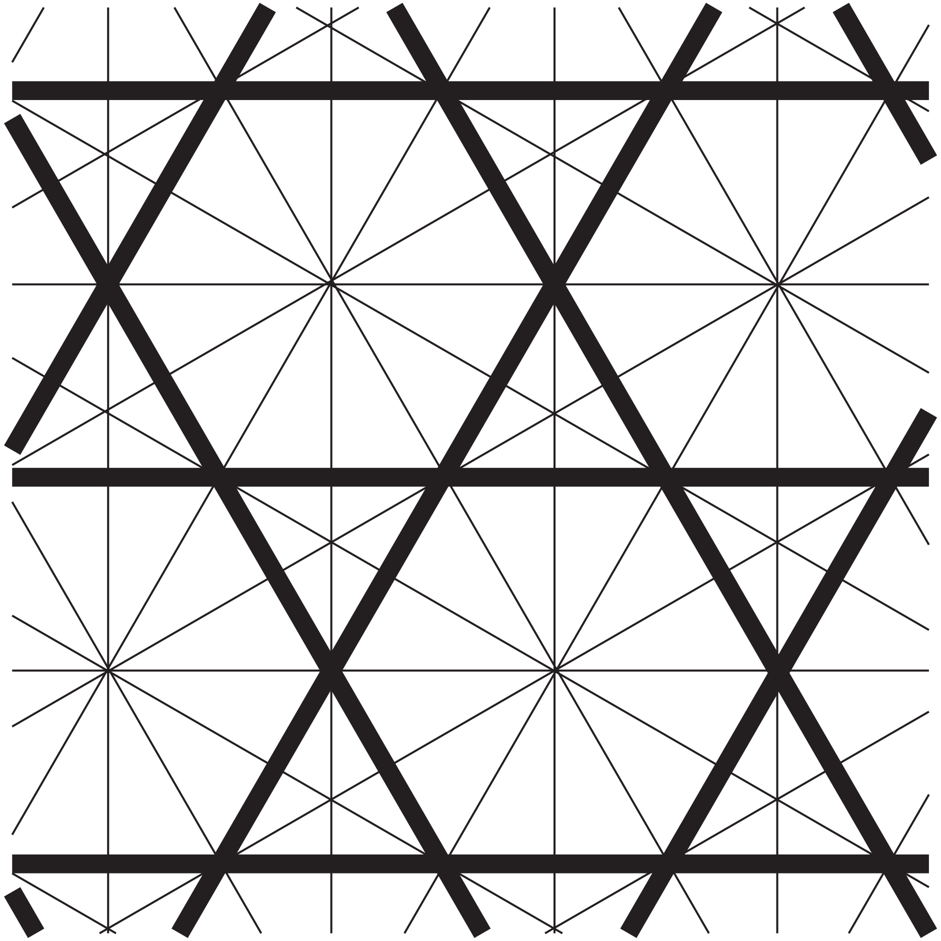
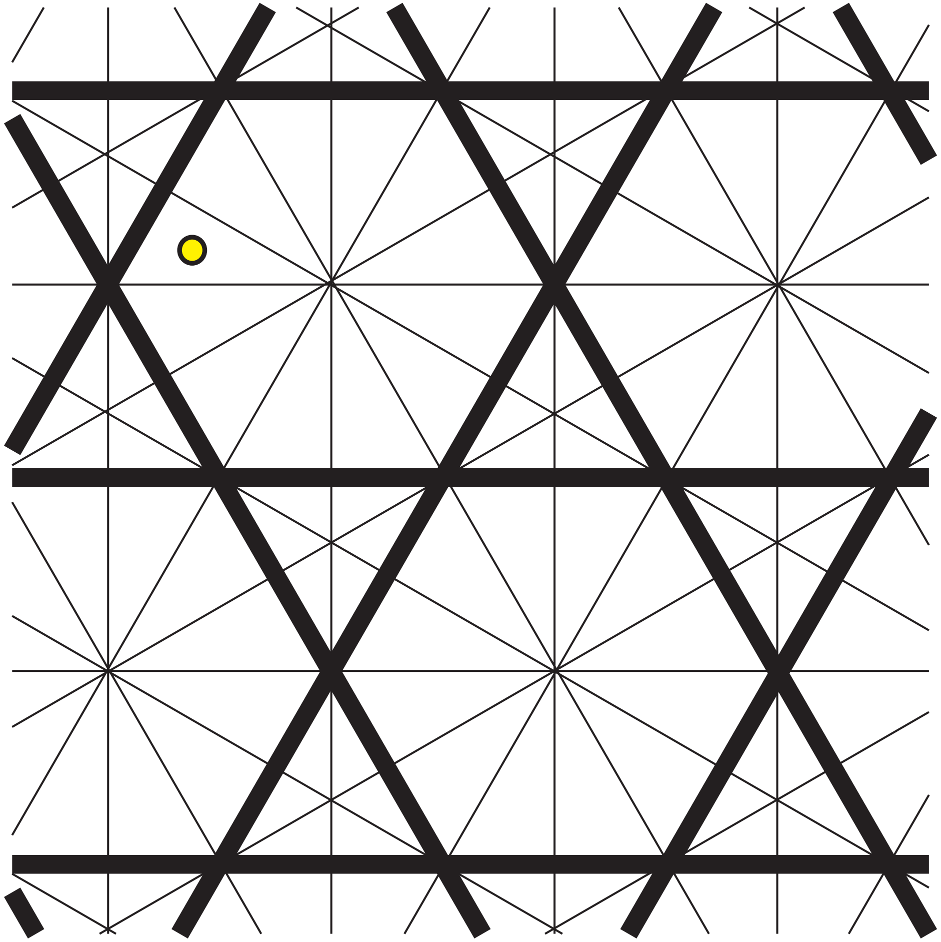


Stabilizer of a flag



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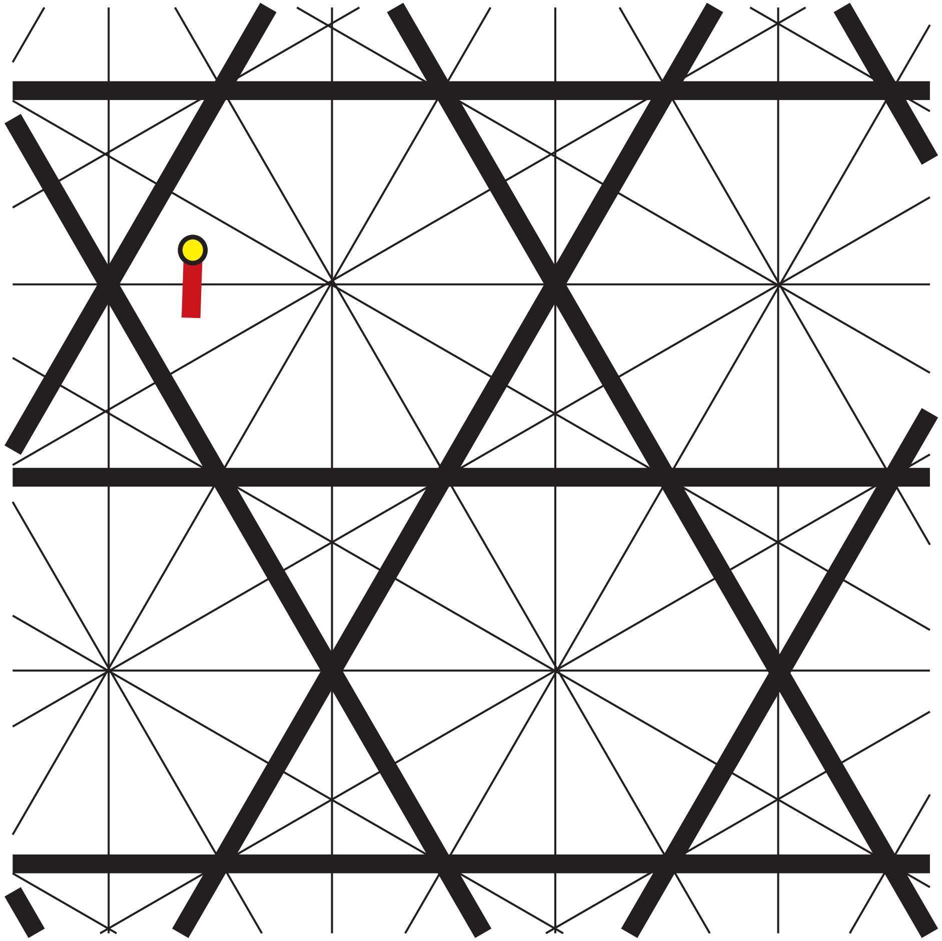

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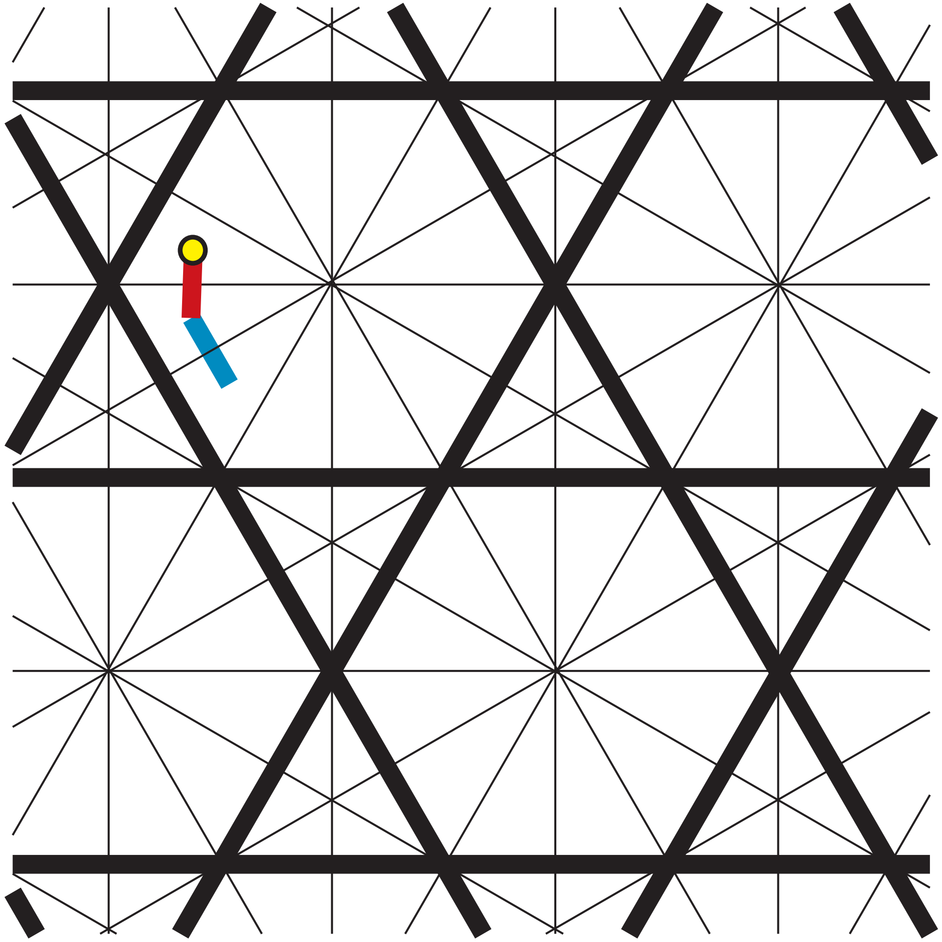

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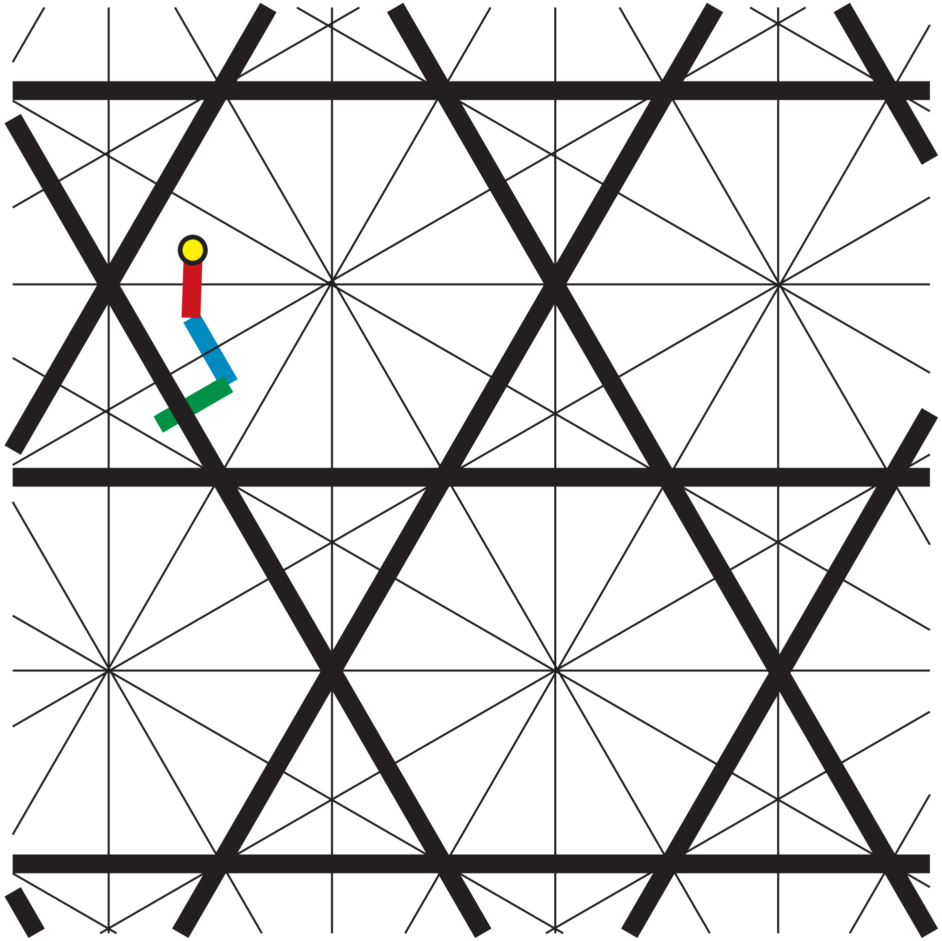

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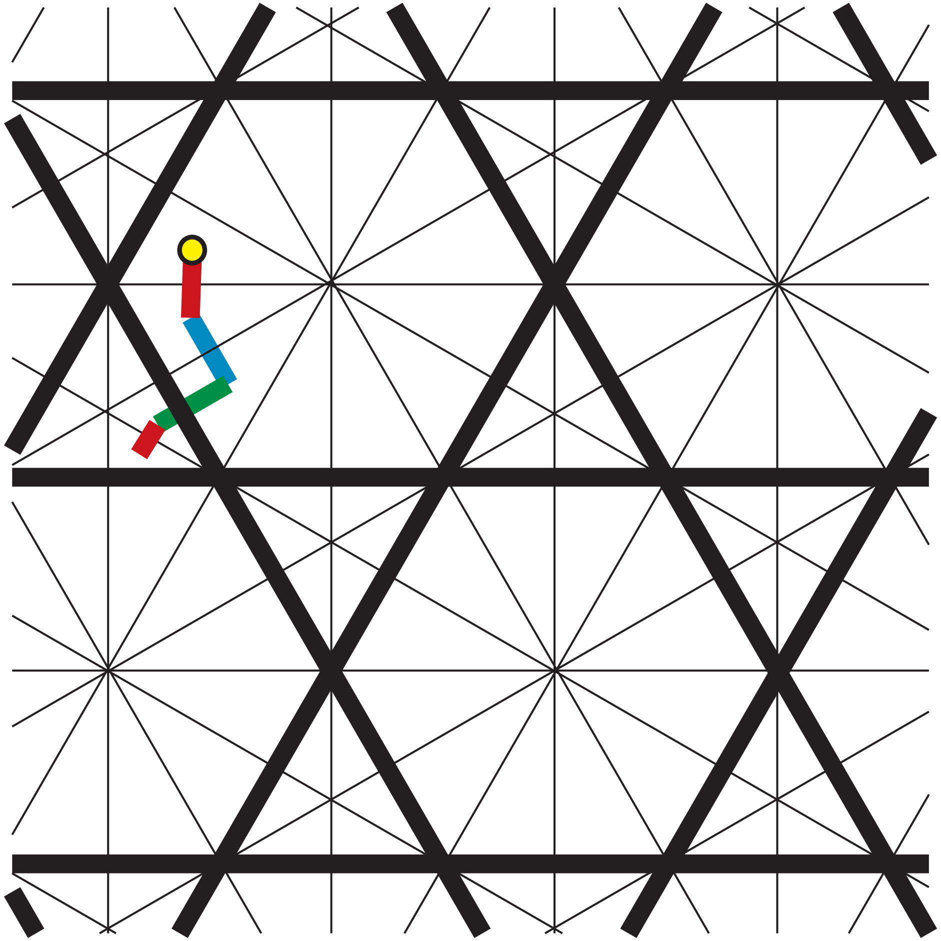

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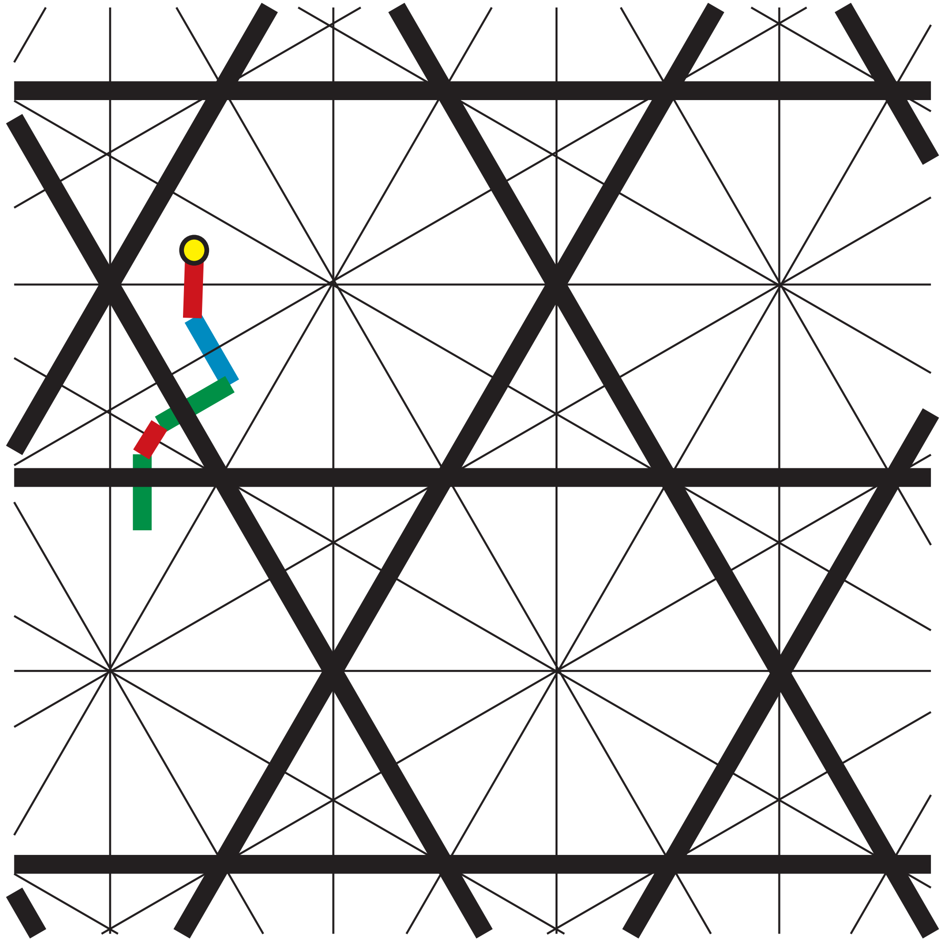

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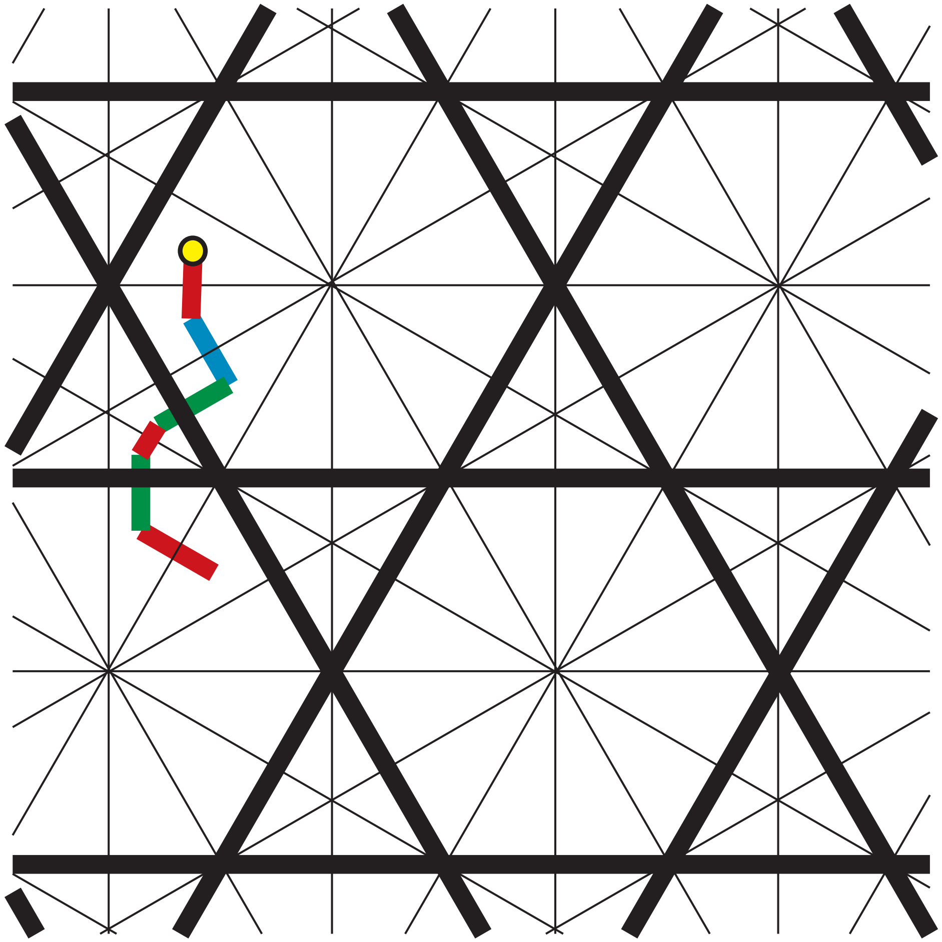

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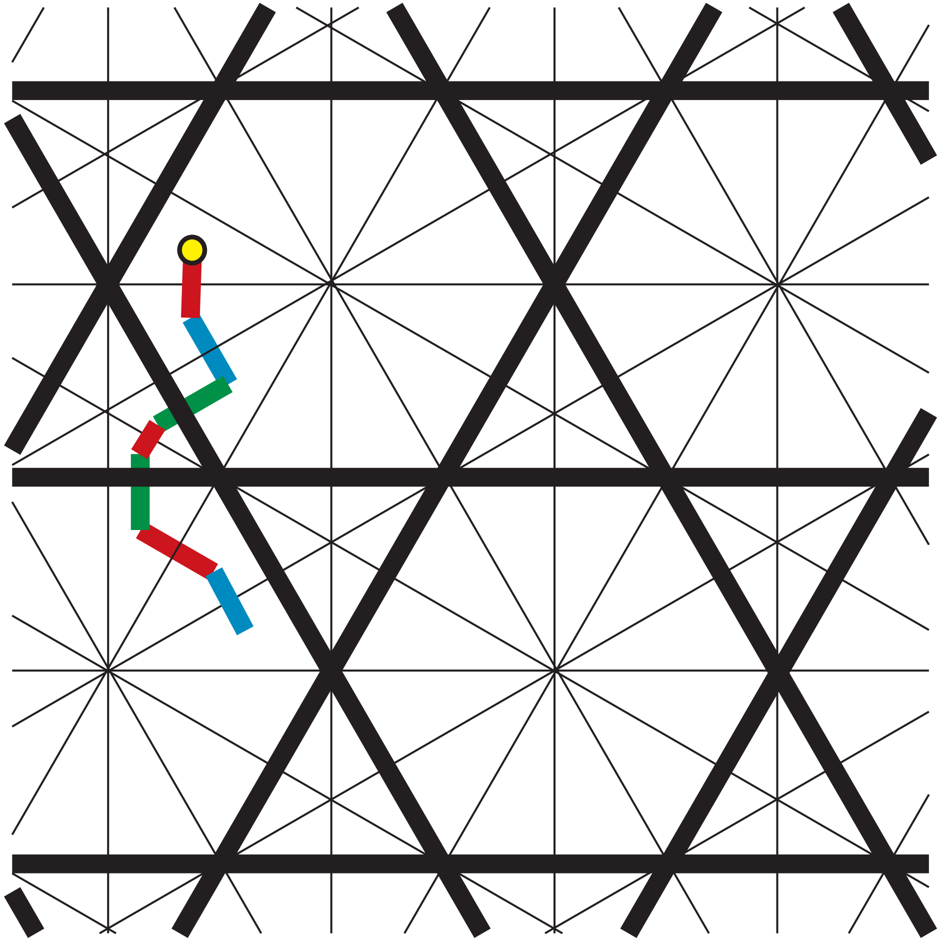

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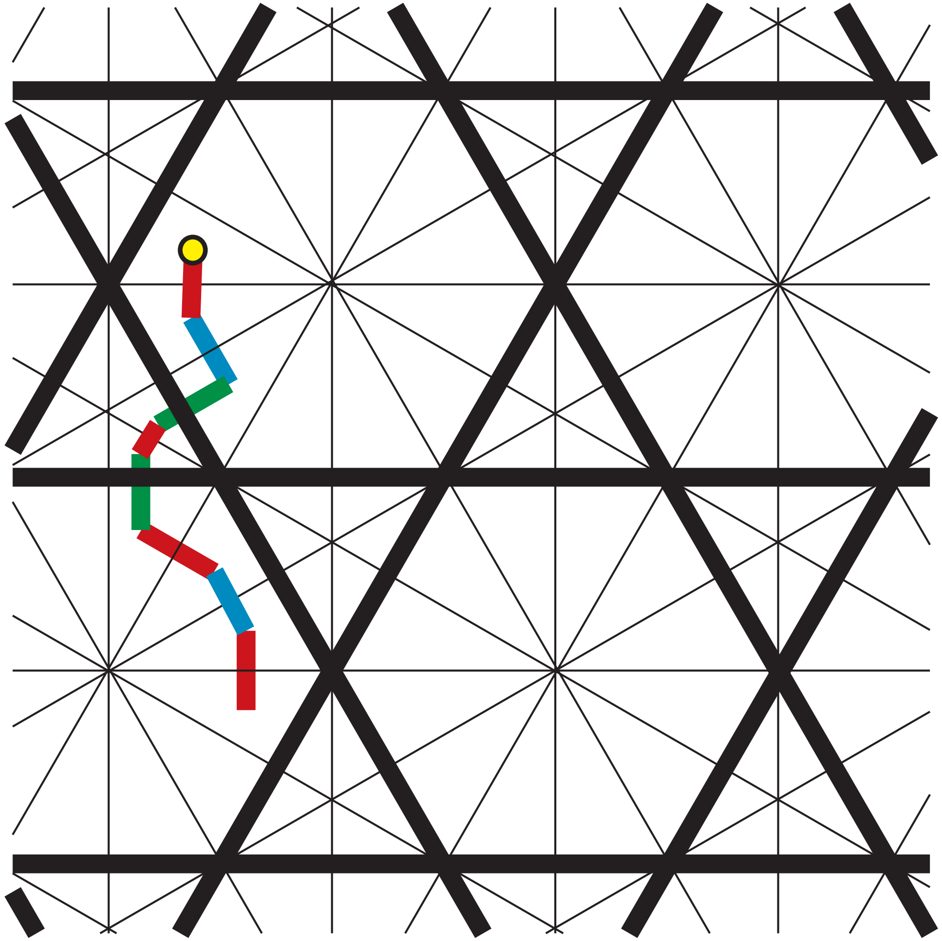

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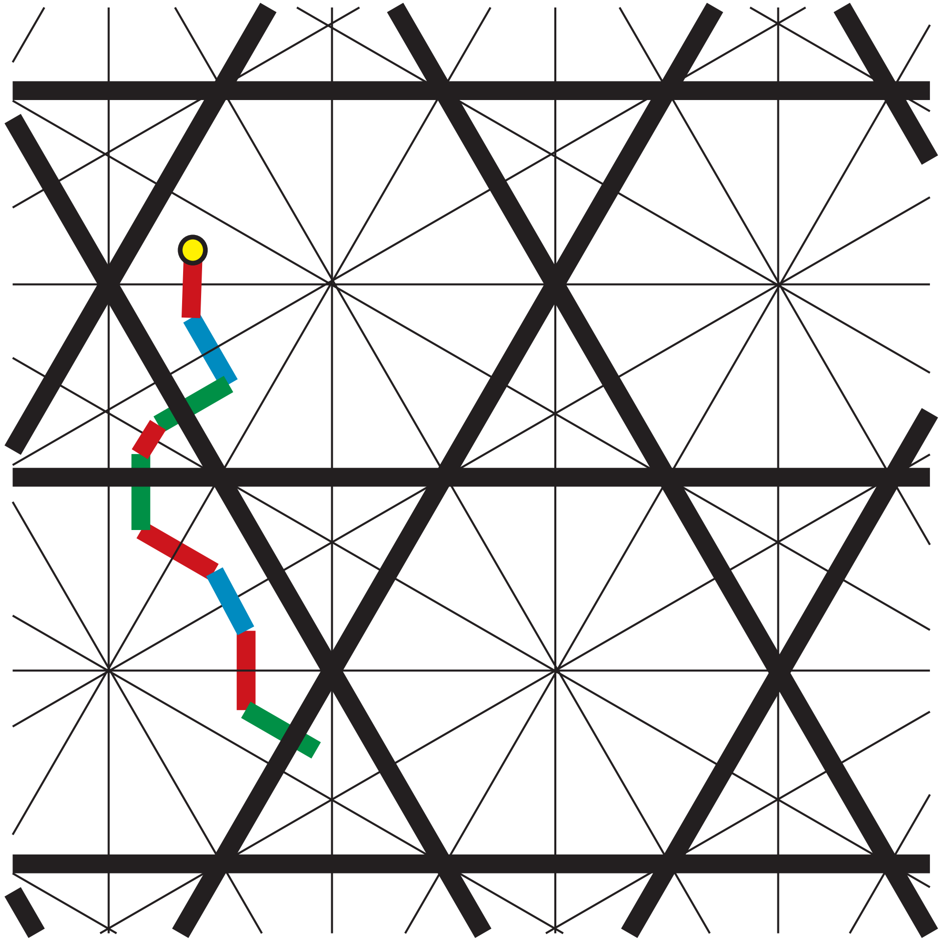

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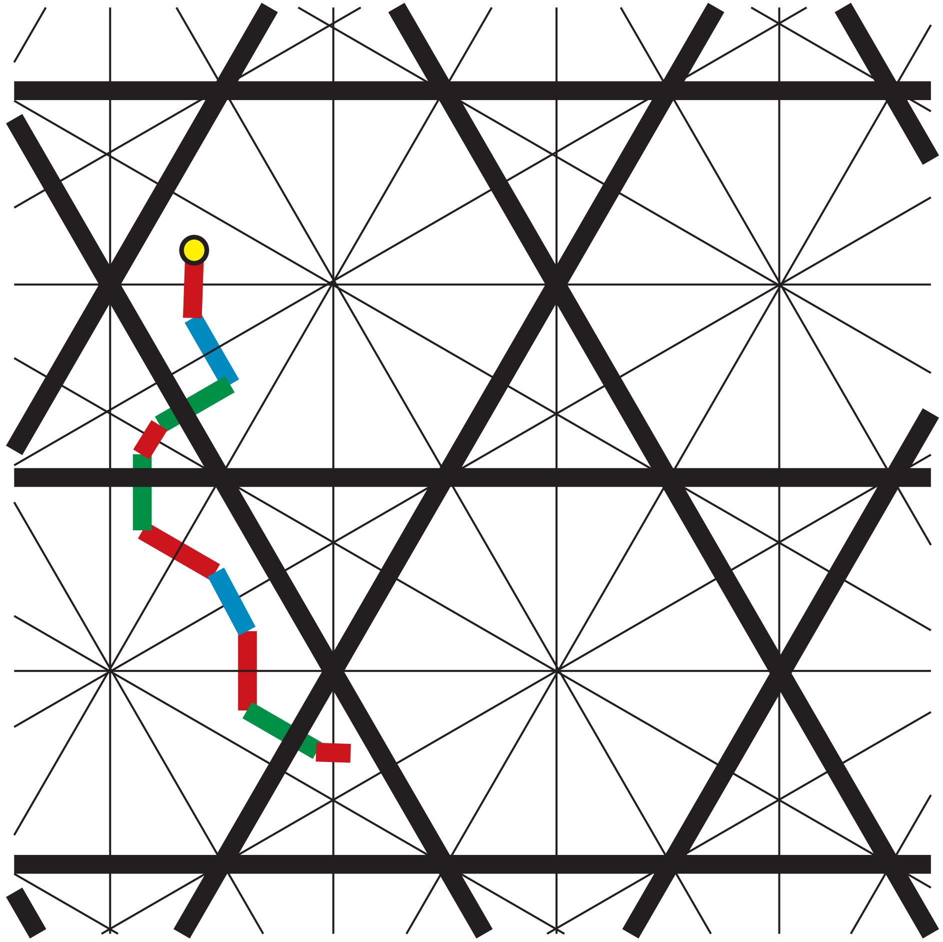

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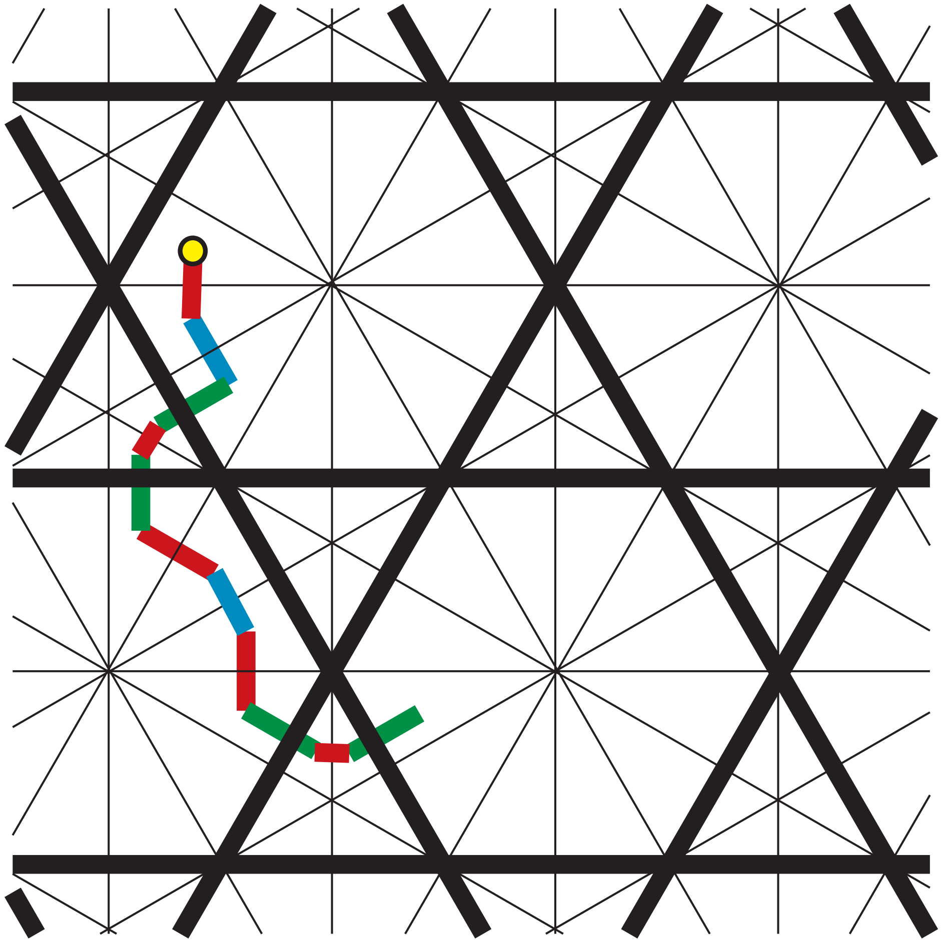

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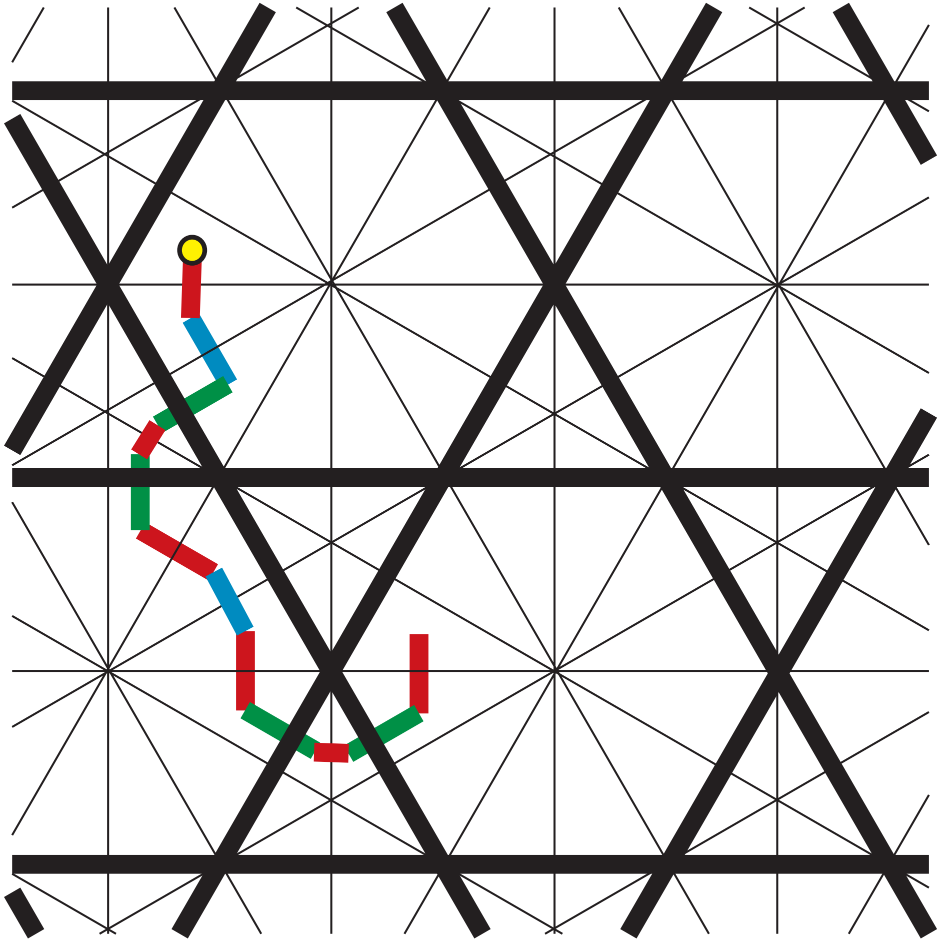

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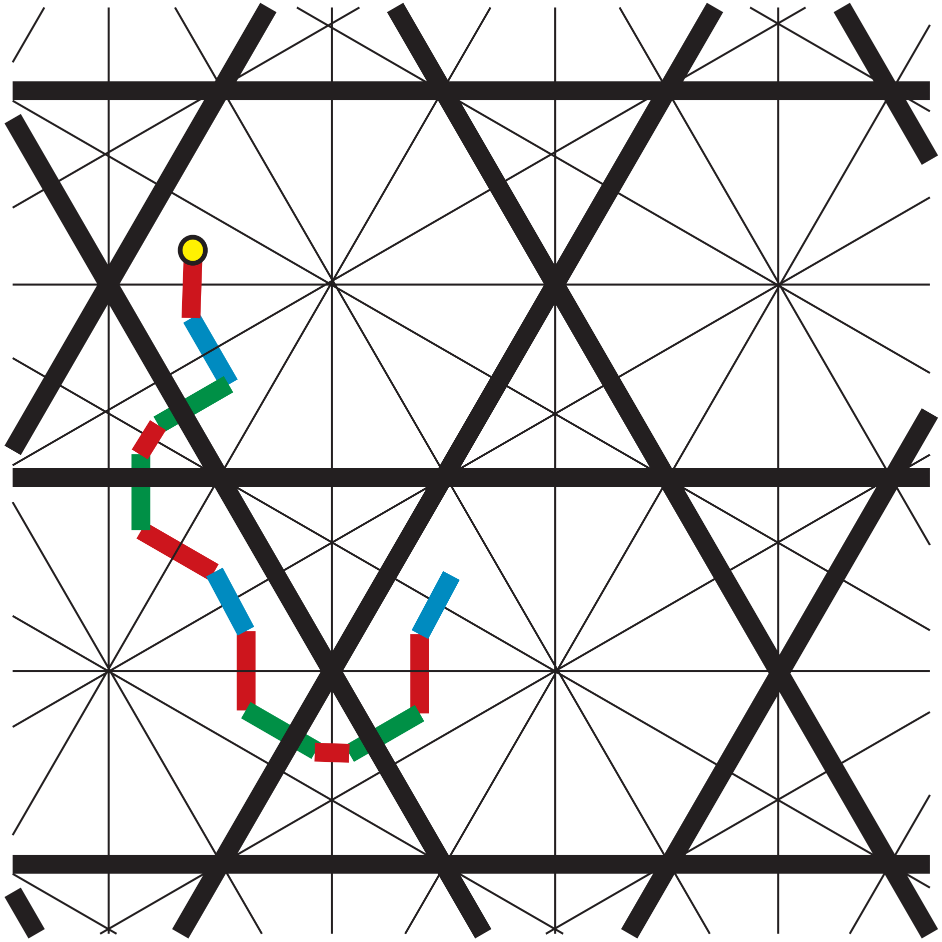

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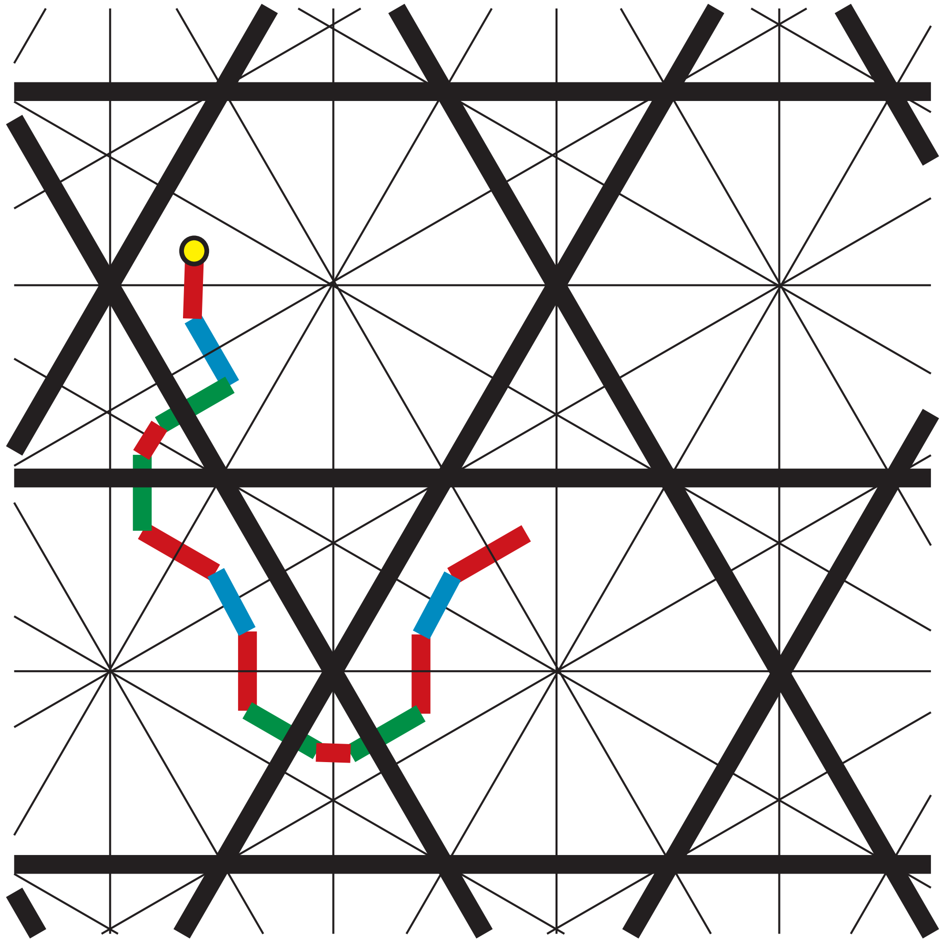

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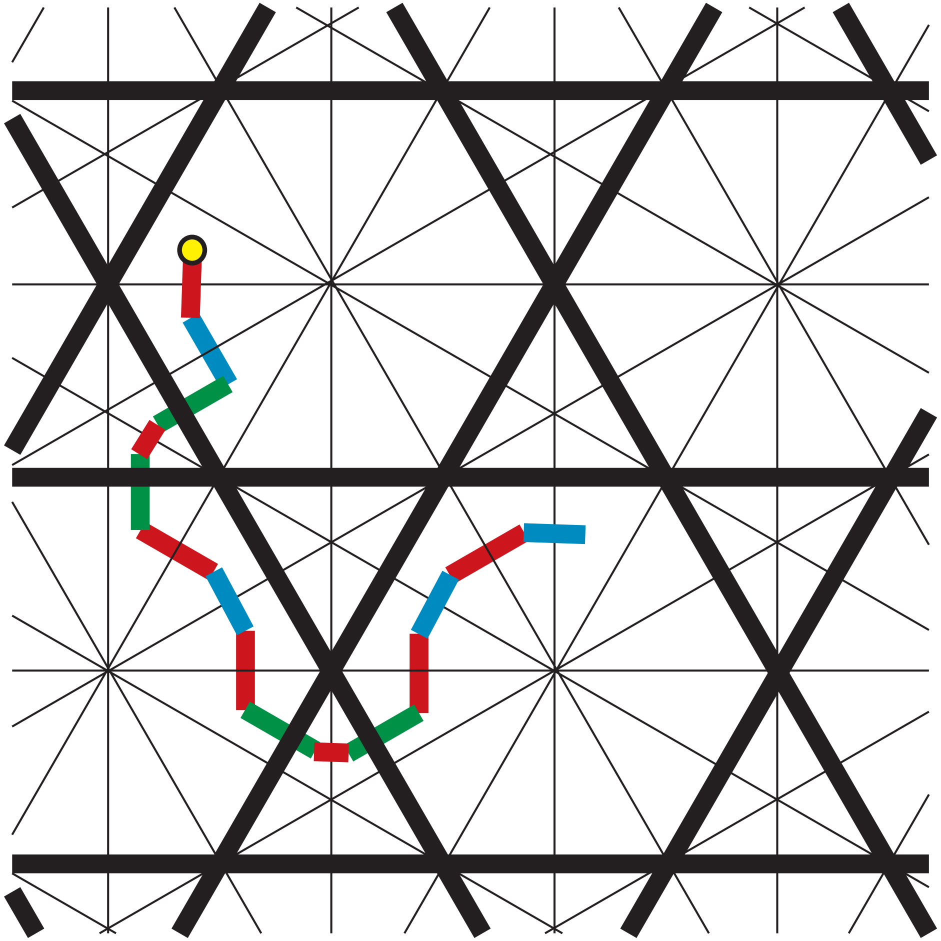

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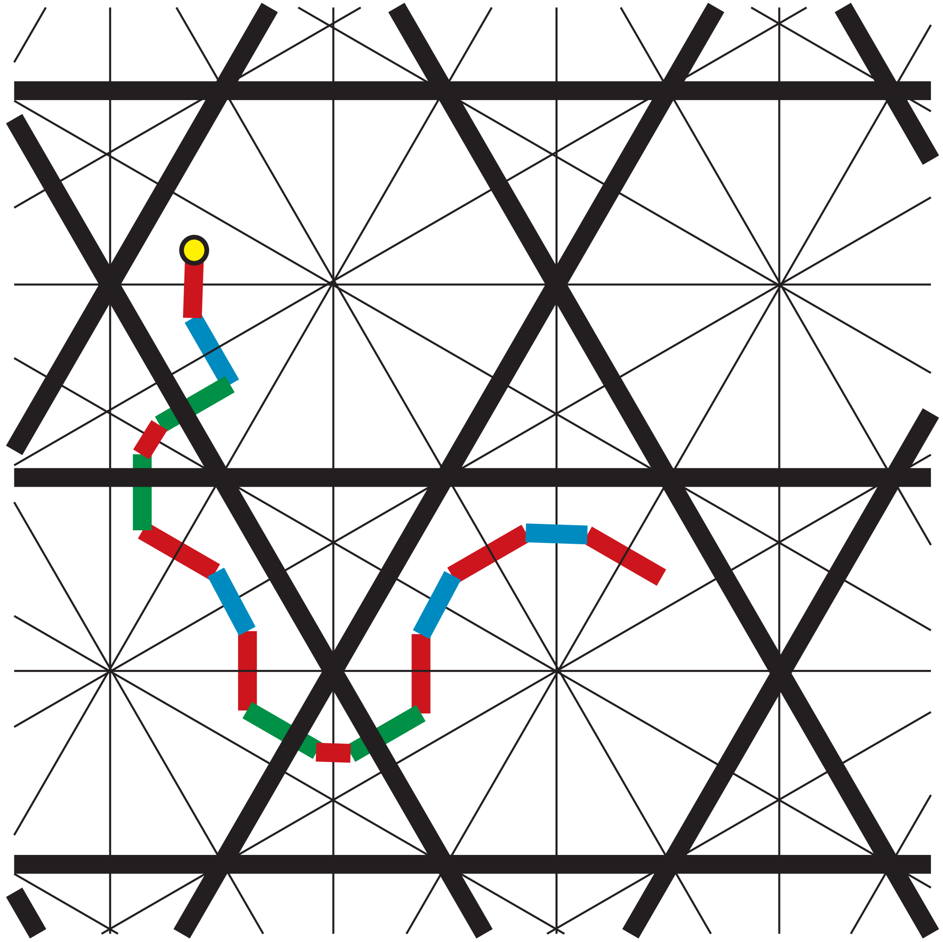

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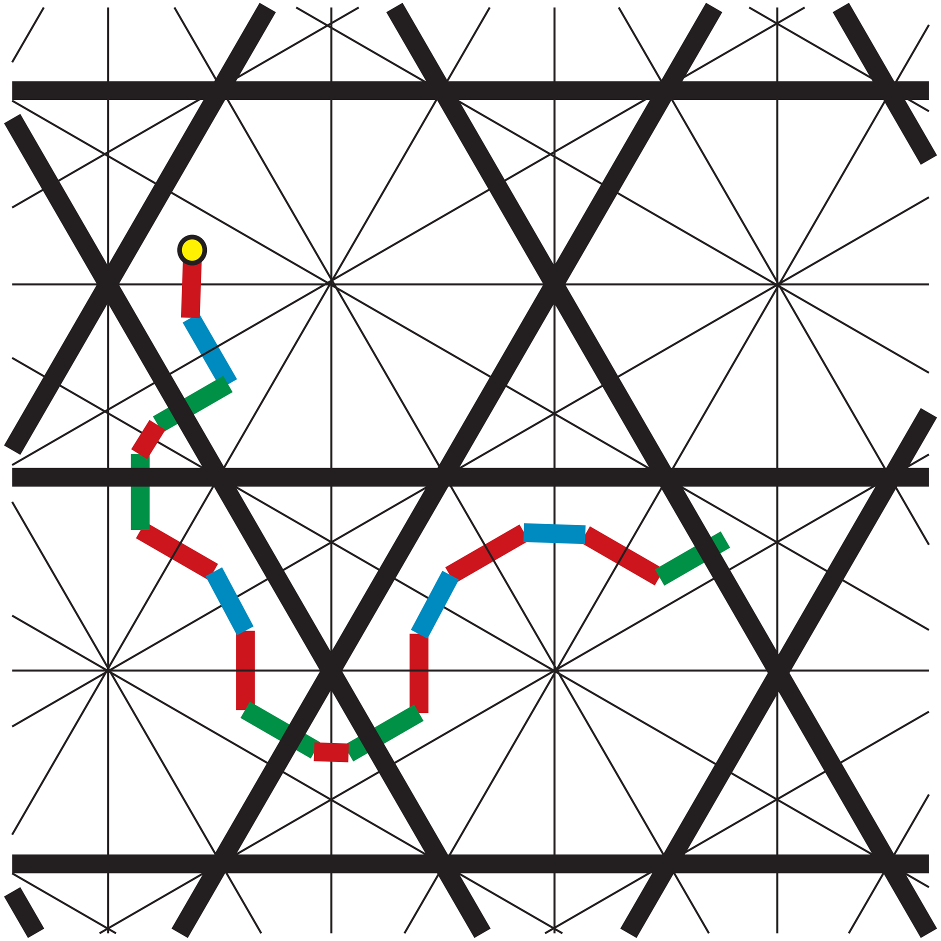

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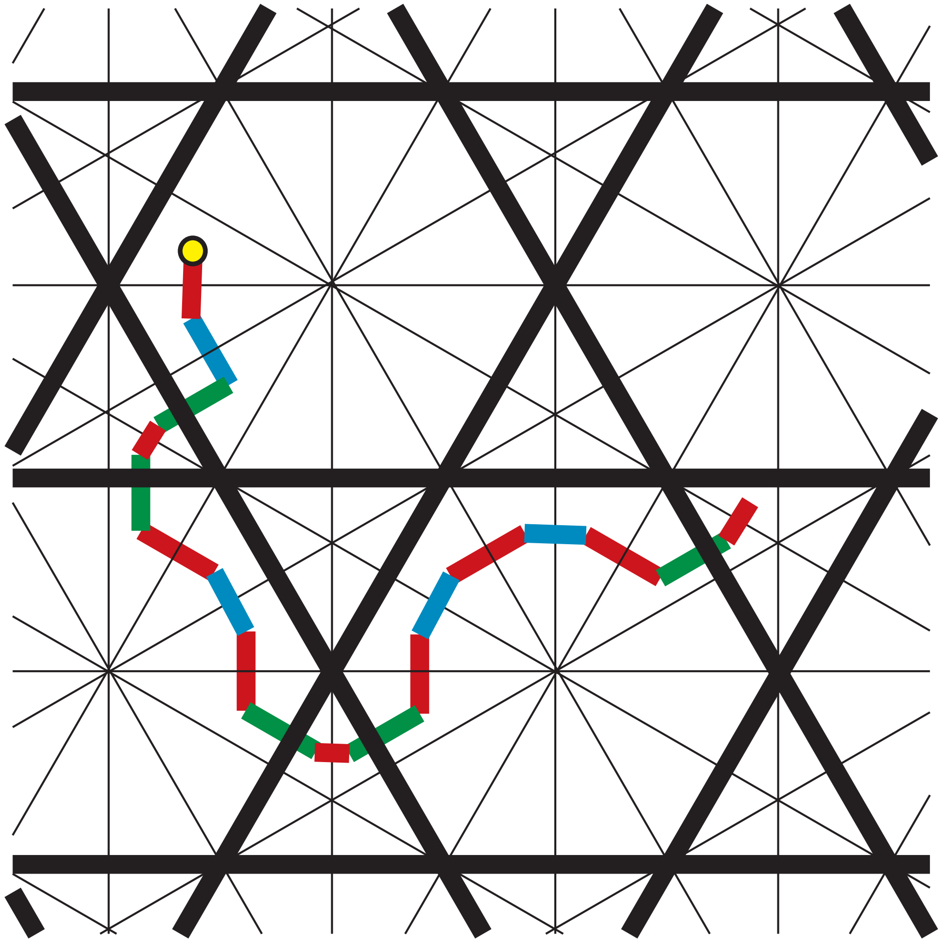

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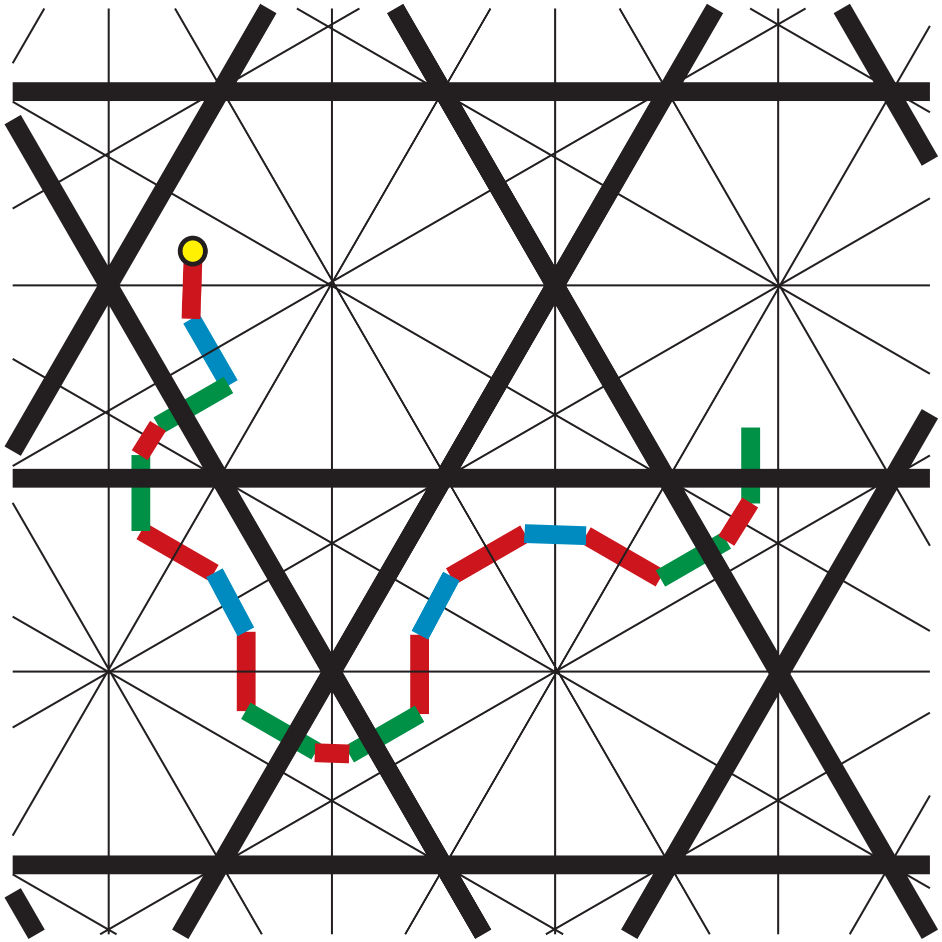

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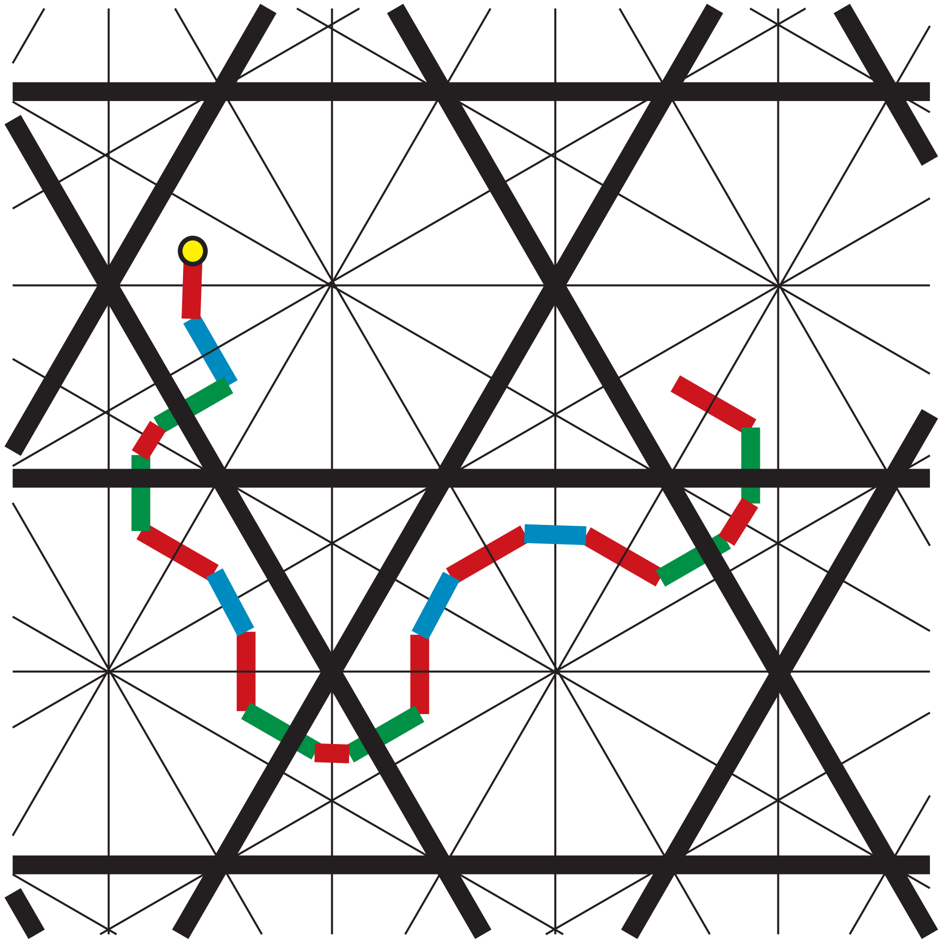

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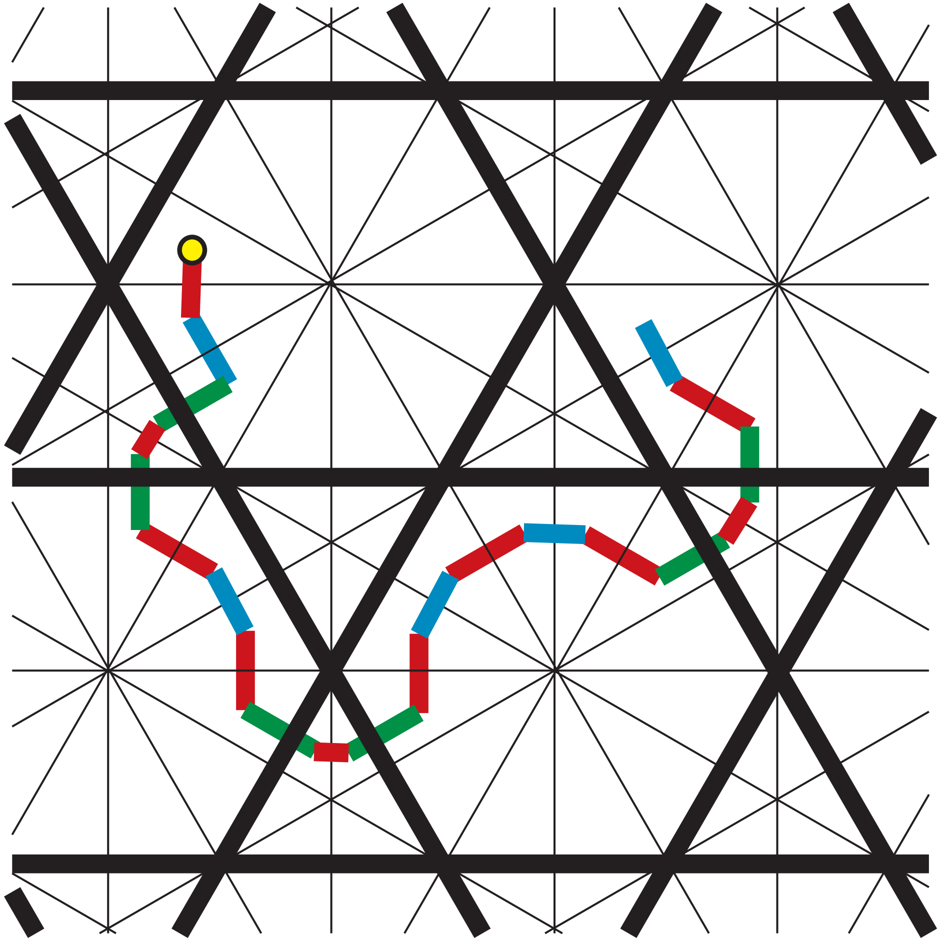

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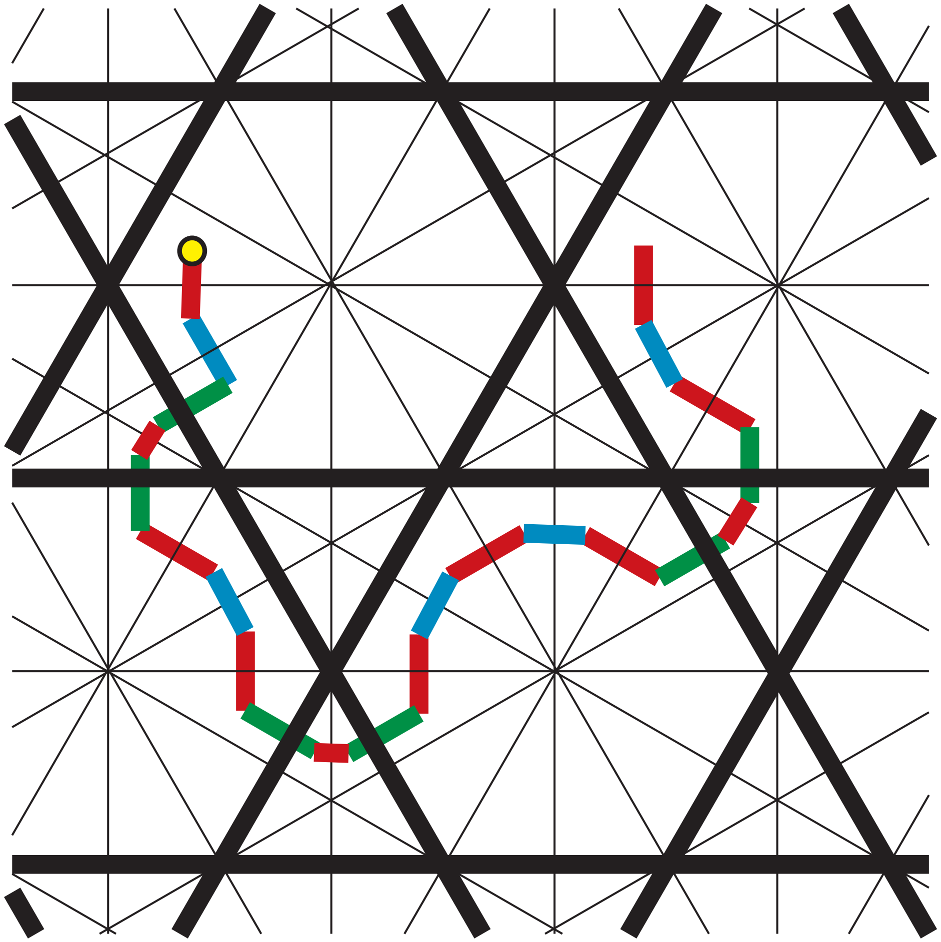

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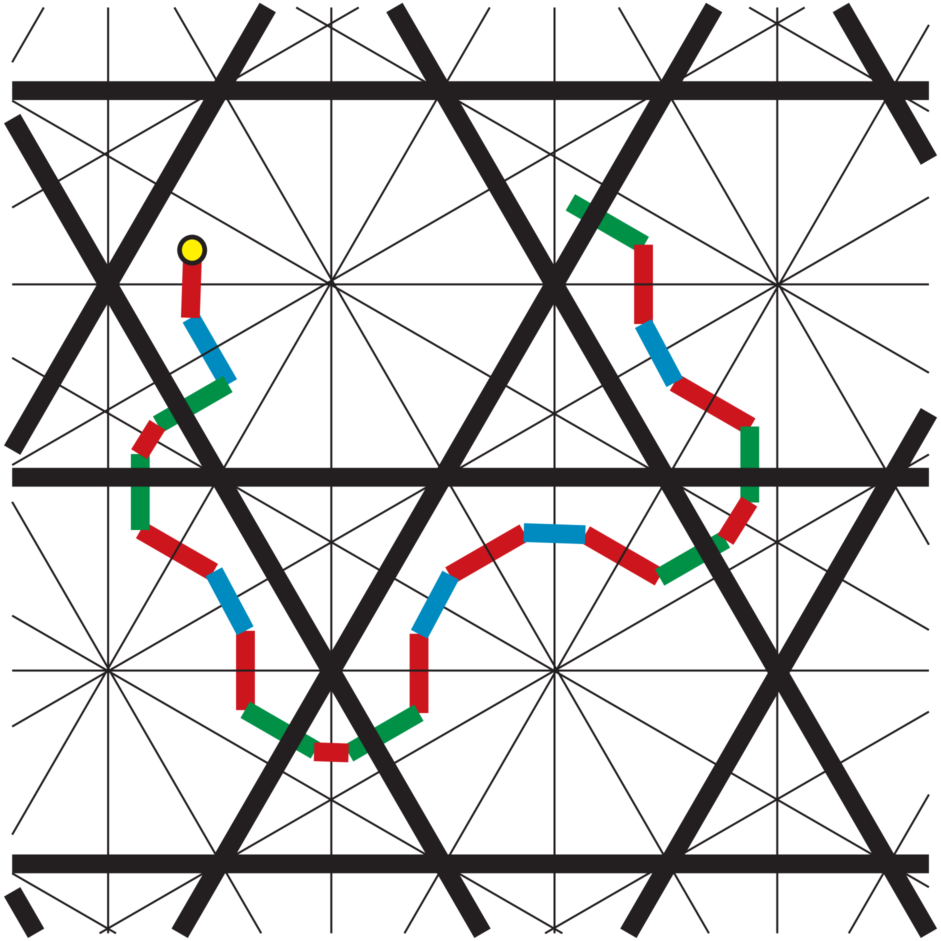

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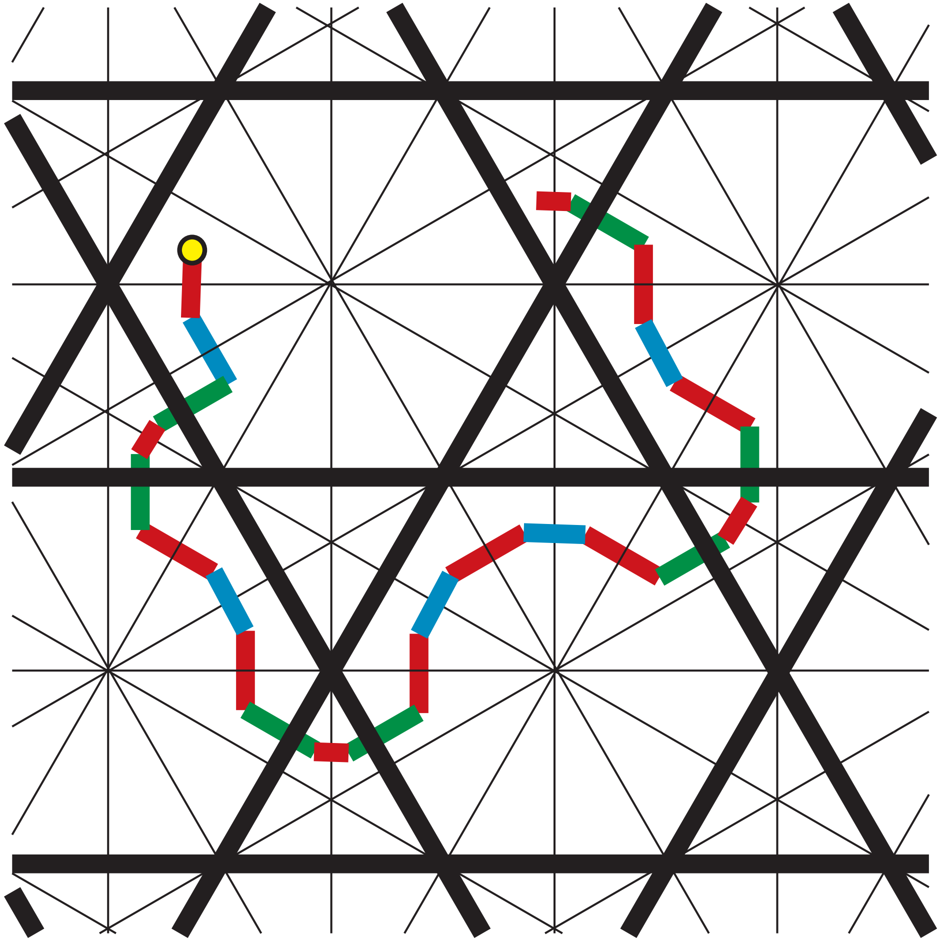

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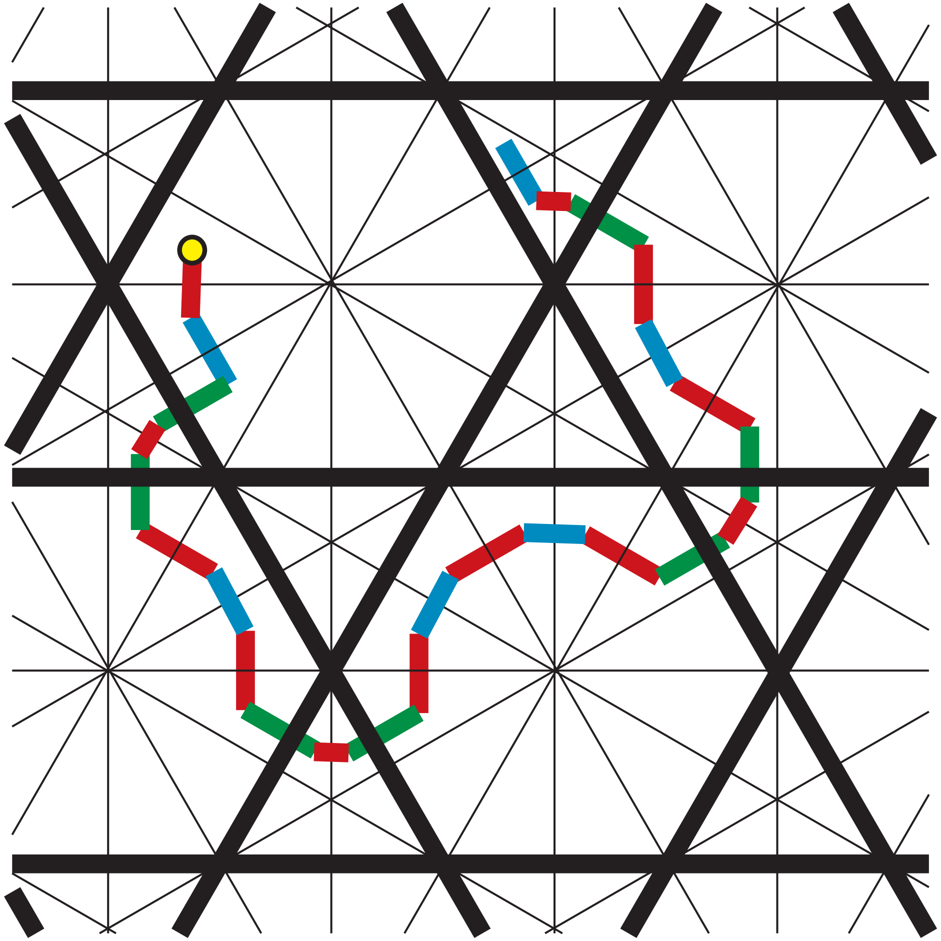

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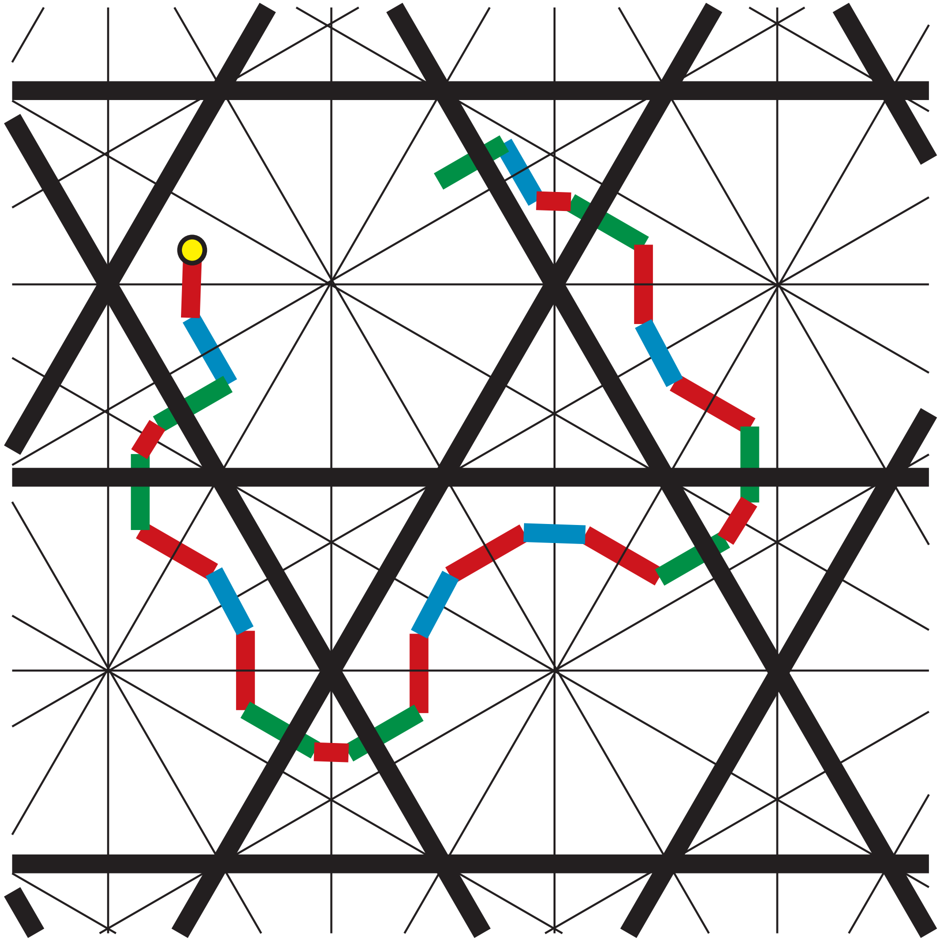

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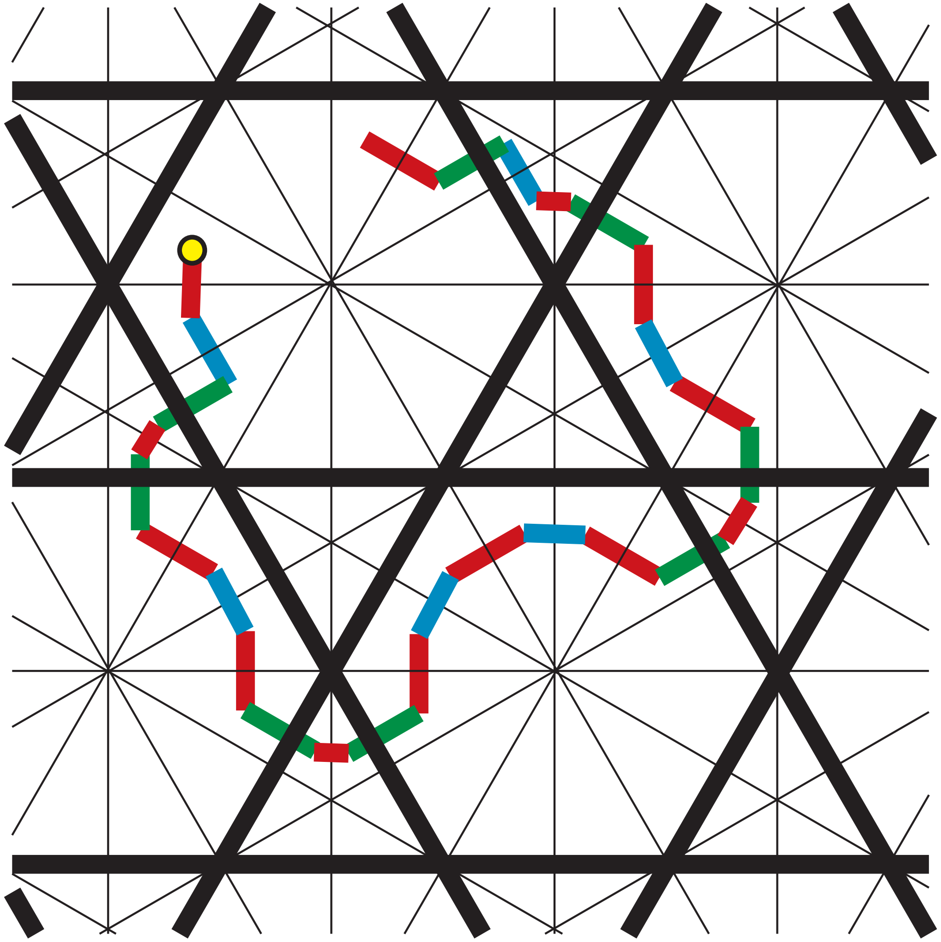

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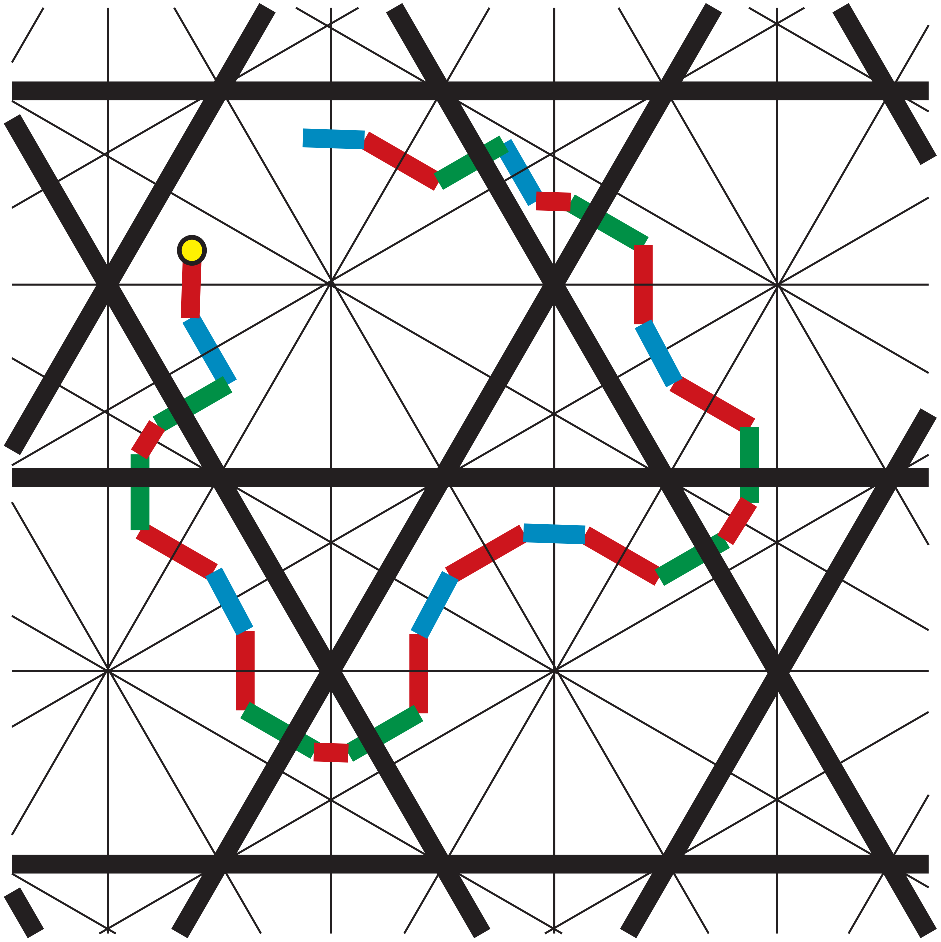

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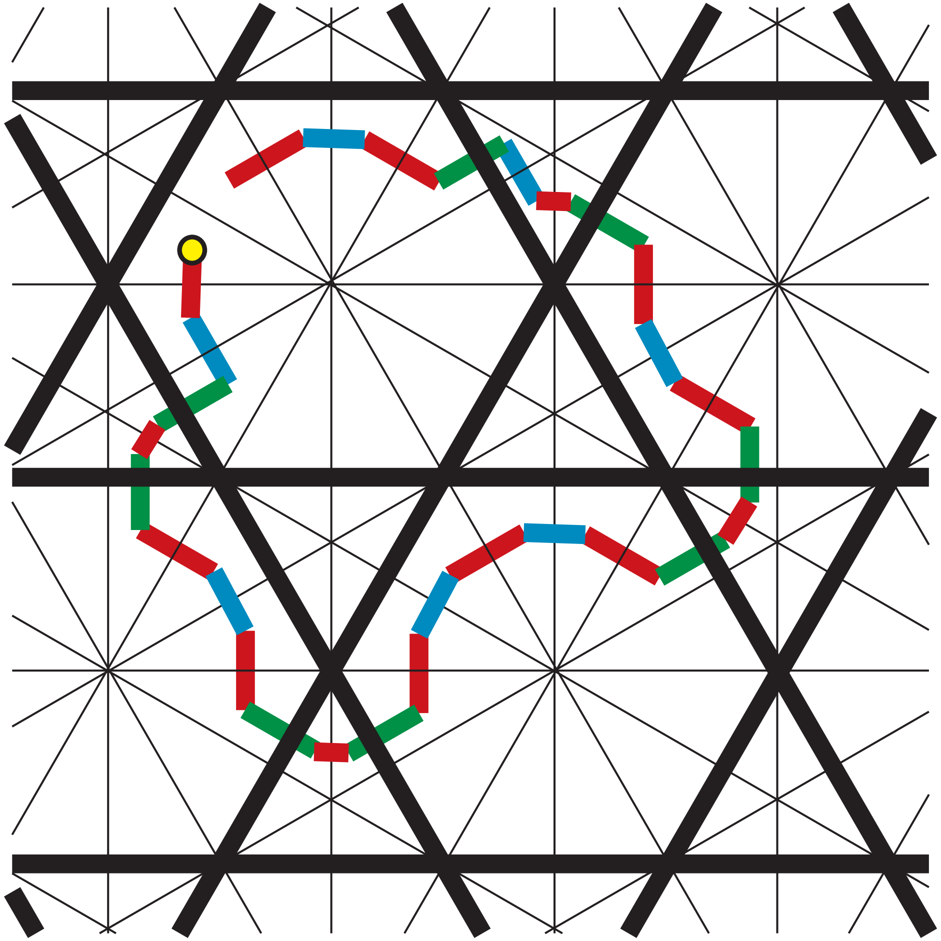

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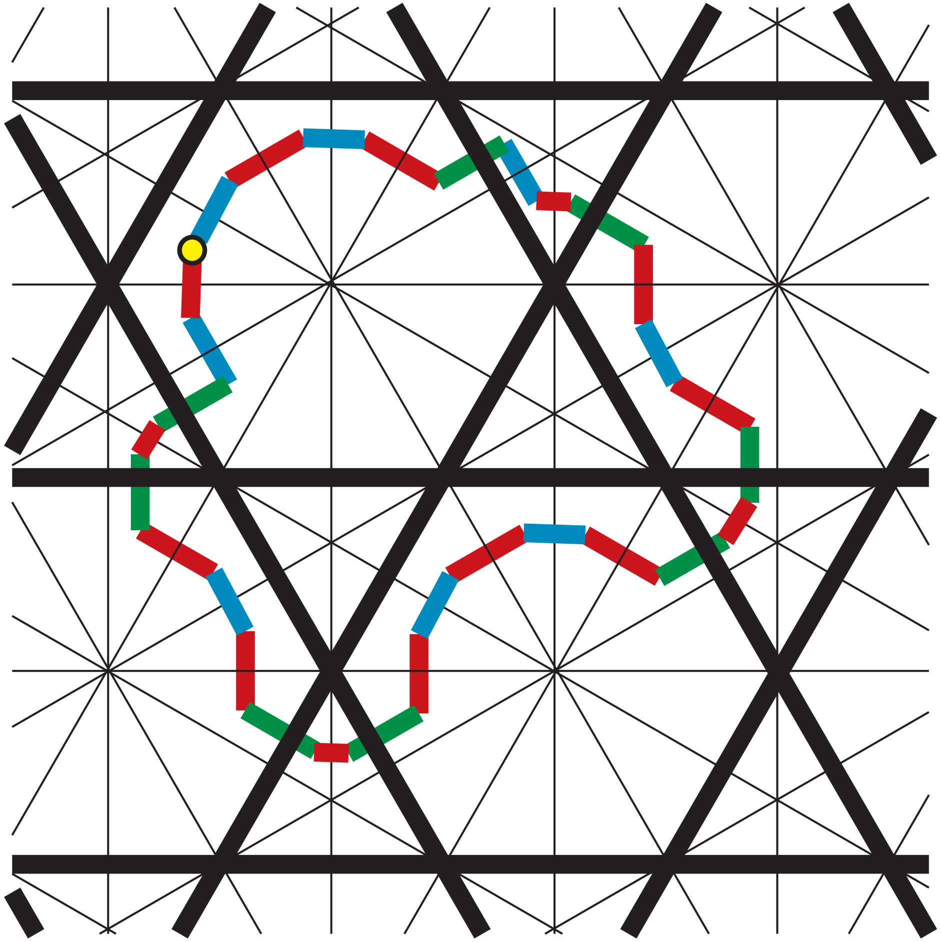

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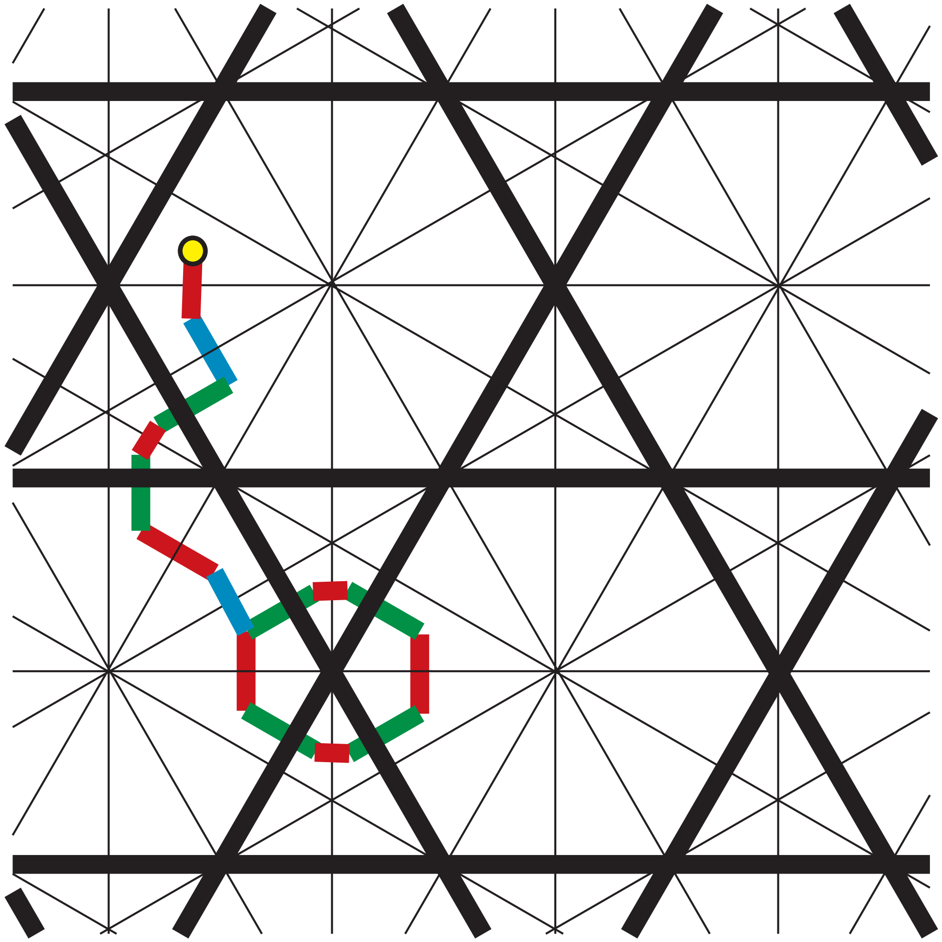

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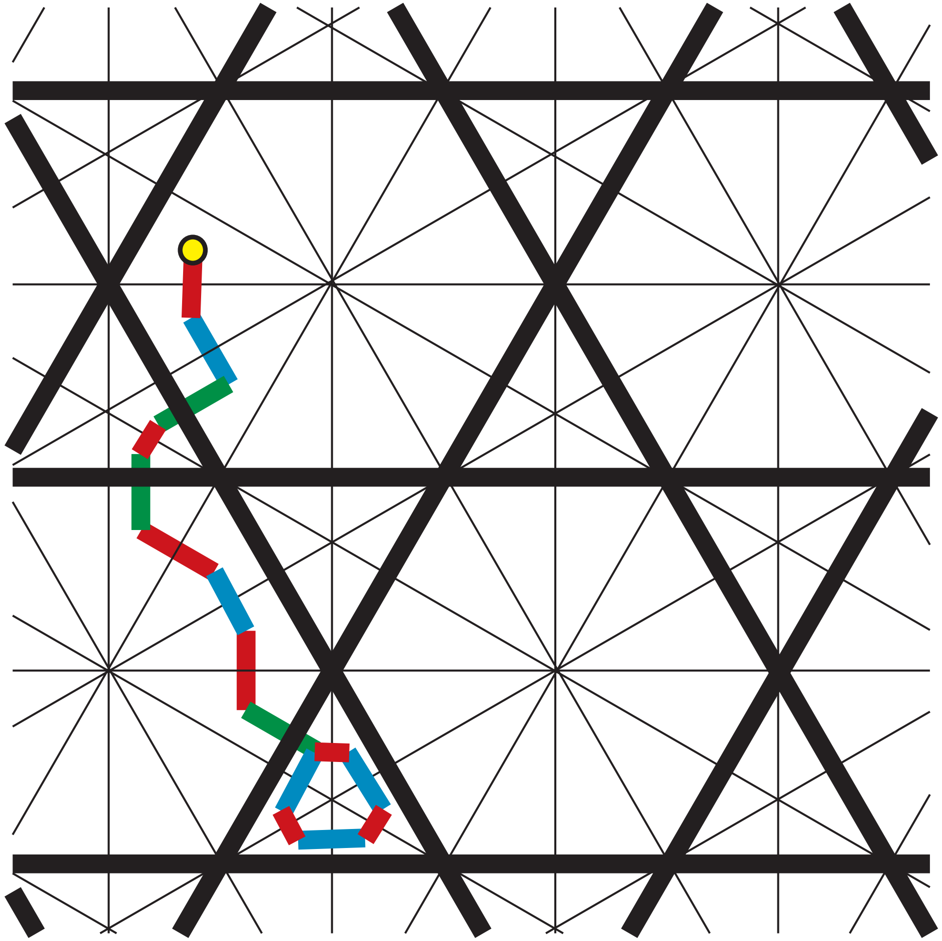

Properties of covers of uniform tilin

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- The minimal coverings are not the hyperbolic tessellations

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- The minimal coverings are not the hyperbolic tessellations
- The index of all regular covers is infinite

The tessellation $3.6.3.6$

MINIMAL REGULAR COVER

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MINIMAL REGULAR COVER

The hyperbolic tessellation $\{6, 4\}$ subject to the relations

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$$[(\rho_1\rho_0)^2\rho_1\rho_2]^4$$

The tessellation $3.6.3.6$

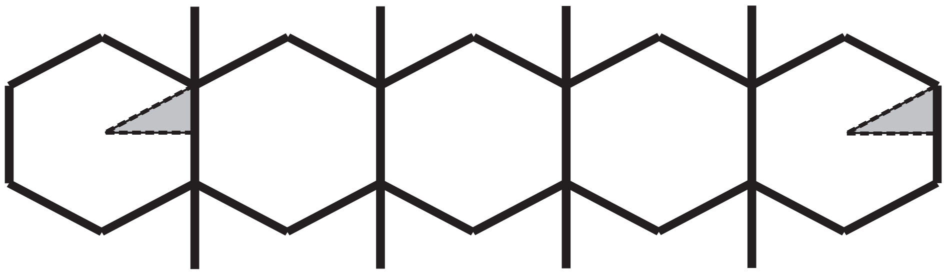
MINIMAL REGULAR COVER

The hyperbolic tessellation $\{6, 4\}$ subject to the relations

$$[(\rho_1\rho_0)^2\rho_1\rho_2]^4$$

$$[(\rho_1\rho_0)^2\rho_2]^6$$

The tessellation $3.6.3.6$



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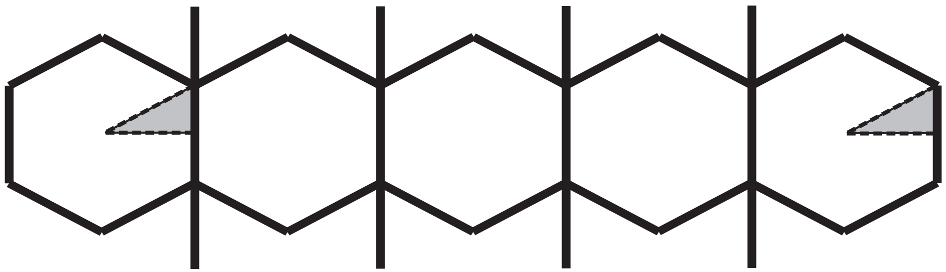

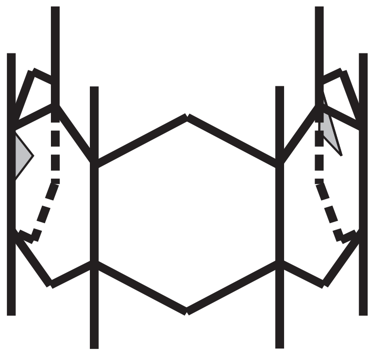

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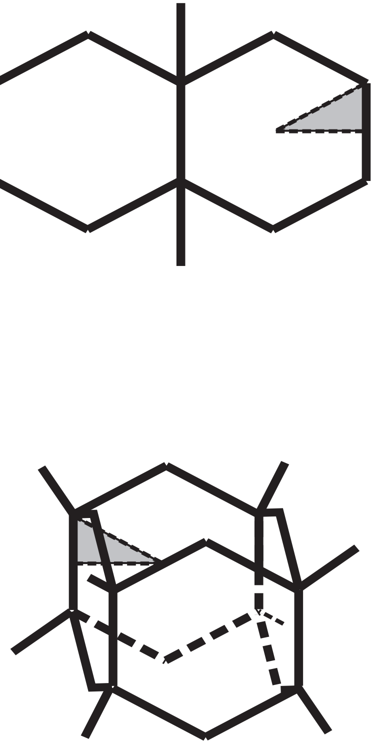

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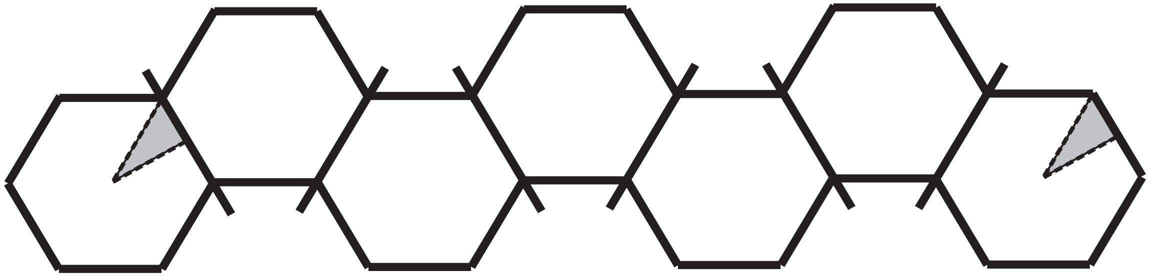
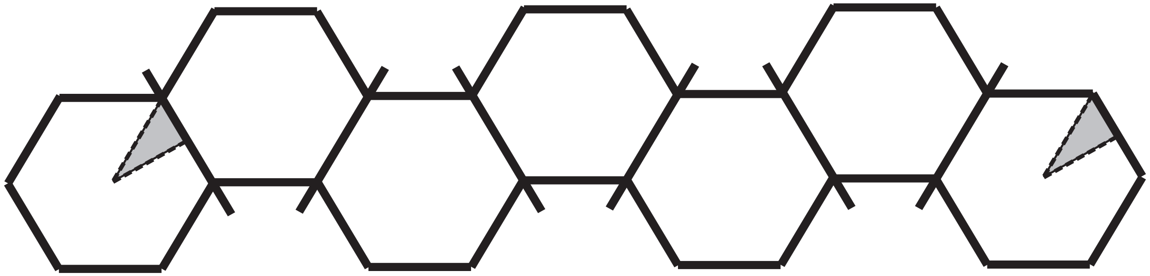
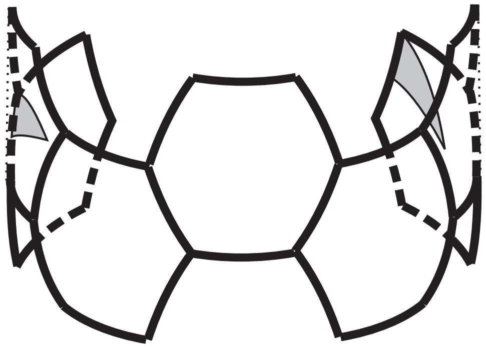
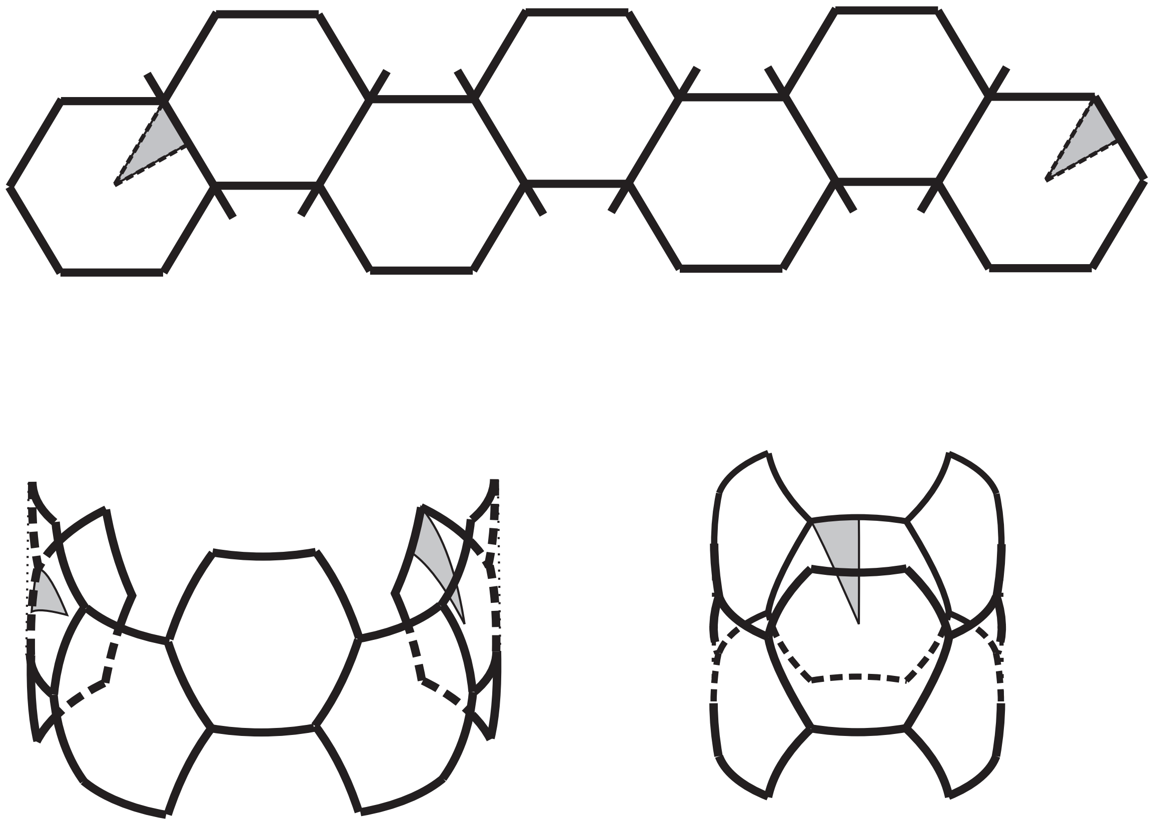


The tessellation $3.6.3.6$





The tessellation 3.6.3.6



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