Finite group actions on Klein bottle

Are all vertex-transitive maps on Klein bottle Cayley?

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June 29, 2009

Outline

Classical results

- Maschke(1896) finite groups with planar Cayley graphs
- Proulx (1978) minimal toroidal graphs
- Tucker (1980) finite number of minimal Cayley graphs for genus $g > 1$

Wormald examples

- infinitely many vertex transitive graphs of genus 2.
- all have symmetric embedding into a Klein bottle
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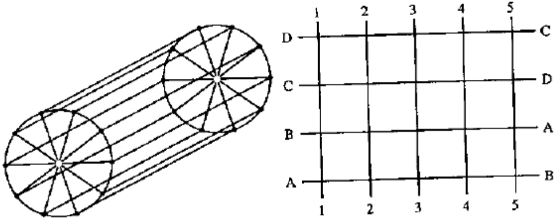

Babai's conjecture

Every sufficiently large vertex-transitive graph on a surface of given genus admits embedding to torus or Klein bottle.

- Thomassen (1991) - topological arguments
- Babai (1991) - group theoretic approach

All large enough graphs embed *symetrically*.

Group actions on torus (S.- to appear)

- presentations of toric groups
- Cayley actions on semiregular tilings
- all vertex-transitive maps on torus are Cayley

Outline

History

Groups acting on Klein bottle

Cayley maps on Klein bottle

KBU groups

wallpaper group is a group with discrete action on plane containing independent translations

KBU group is a wallpaper group admitting *pg* as a normal subgroup.

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Possible KBU groups:

- pg
- pm
- cm
- pgg
- pmg
- pmm
- cmm

Odd glides in KBU groups

odd glide is a glide that is not the product of reflection and a translation

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odd glide is a glide that is not the product of reflection and a translation

Wallpaper group	glide	complement	quotient
pg	P^{2n-1}	$(PQ^{-1})^2$	$\mathbf{Z}_{2(2n-1)}$
cm	$(RS)^{2n-1}$	$RSRS^{-1}$	$\mathbf{Z}_{2(2n-1)}$
pmg	$(T_1R)^{2n-1}$	$(T_1T_2)^2$	$D_{4(2n-1)}$
pgg	O^{2n-1}	P^2	$D_{2(2n-1)}$
cmm	$(R_1T)^{2n-1}$	$(RT)^2$	$D_{4(2n-1)}$

Even glides in KBU groups

even glide is a glide that is not an odd glide

Even glides in KBU groups

even glide is a glide that is not an odd glide

Wallpaper group	glide	complement	quotient
pm	RX^n	Y^2	$\mathbf{Z}_2 \times \mathbf{Z}_{2n}$
cm	$R(RS)^{2n}$	$RSRS^{-1}$	$\mathbf{Z}_{4n}$
pmm	RY^n	X	$\mathbf{Z}_2 \times D_{4n}$
pmg	$R(T_1T_2)^n$	$(T_1R)^2$	D_{4n}
cmm	$R_1(TR_2)^{2n}$	$(TR_1)^2$	D_{8n}

Outline

Babai's families of vertex-transitive Cayley maps

Every family is given by a pair (*glide*, *orthogonal translation*) acting on a semiregular tiling of the plane.

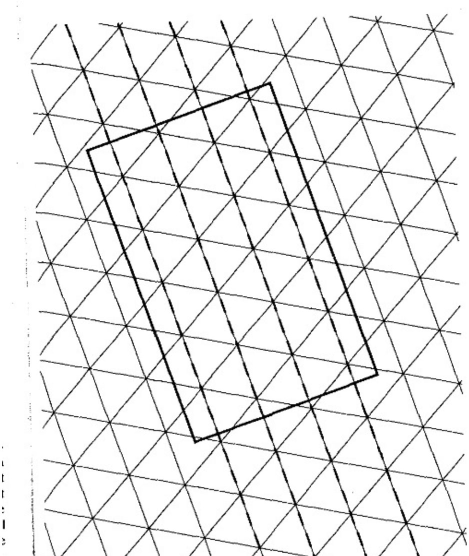
Babai's families of vertex-transitive Cayley maps

Every family is given by a pair (*glide*, *orthogonal translation*) acting on a semiregular tiling of the plane.

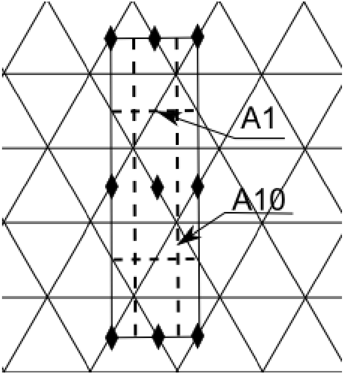
- 13 4-line families
- 5 2-line families
- 1 1-line family

Family $A1$

odd parity

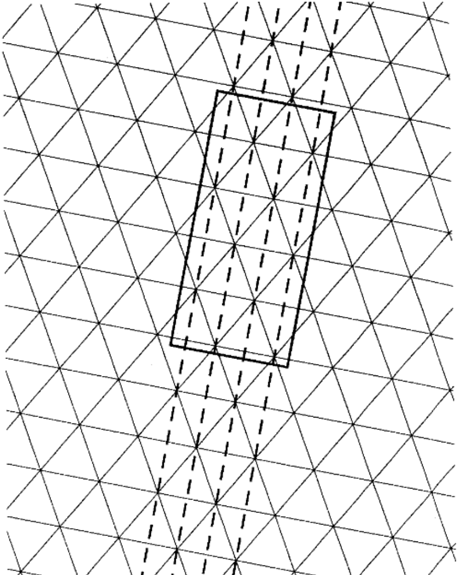


$A1$ - as Cayley map of quotient of pgg group

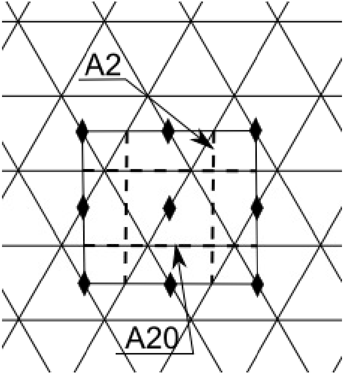


Family $A2$

odd parity

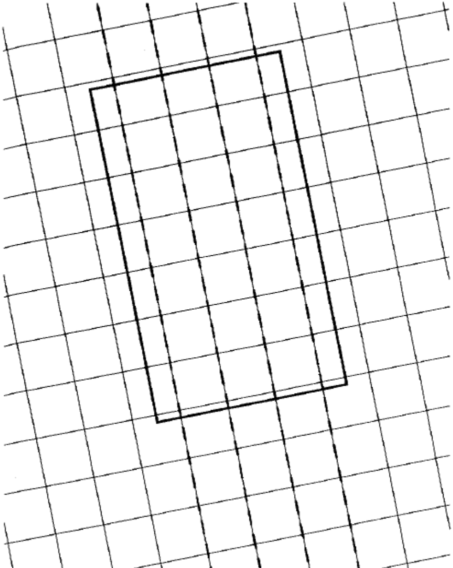


$A2$ - as Cayley map of quotient of pgg group

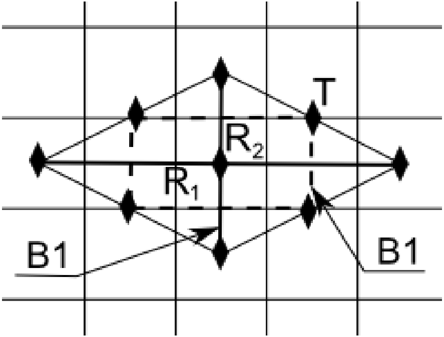


Family $B1$ - Wormald graphs

even and *odd* parity



$B1$ - as Cayley map of quotient of *cmm* group



Family $F1$ - truncated square tiling

even parity

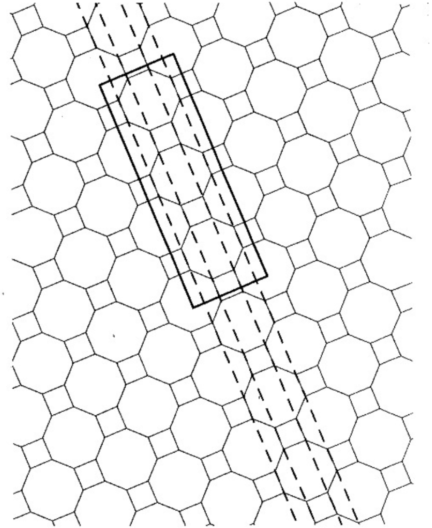
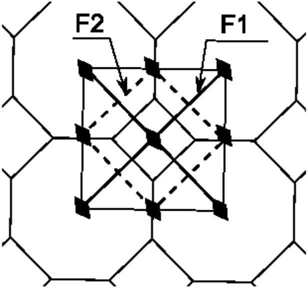


FIGURE 18. $F1$: $(4 \cdot 8^2)$

Family $F1$ - as quotient of cmm



Family $F2$ - truncated square tiling

odd parity

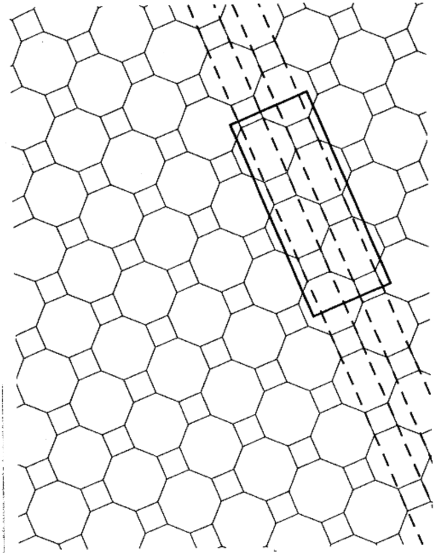


FIGURE 19. $F2$: $(4 \cdot 8^2)$

$C10$, $C3$ - as Cayley map of quotient of pgg group