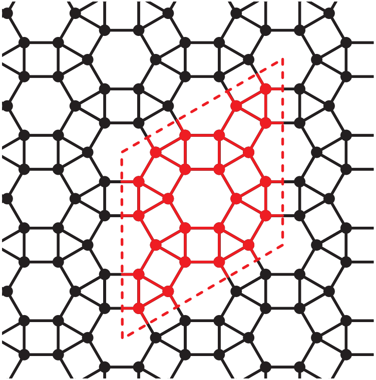
Group actions on orientable surfaces

Ján Karabáš and Roman Nedela

Mathematical Institute, Slovak Academy of Sciences
Matej Bel University

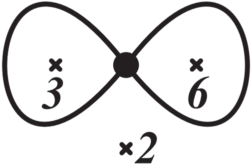
ATCAGC 2013, Bovec, Slovenia

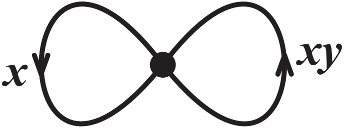
Example: A toroidal map of local type $(3.4.6.4)$



A map defined from the universal cover, the parameters determining the fundamental region are $a = 2, b = 0$

Example: The same map defined as a lift of its quotient





$\bar{\mathbf{M}} \hookrightarrow (0; \{2, 3, 6\})$

$$\mathrm{Aut}^+(\mathbf{M}) = \langle x, y \mid x^3 = y^2 = (xy)^6 = 1, (x^{-1}yxy)^2(xy^{-1}xy)^0 = 1\rangle$$

Map is a drawing of a finite connected graph on a surface without edge-crossings.

More formally: *(topological) Map is a cell-decomposition of a surface*.

Surfaces: closed, connected, orientable.

Combinatorial map: $\mathbf{M} = (D; R, L)$, D - finite set, $R, L \in \mathrm{Sym}(D)$, $\langle R, L\rangle$ is transitive on D, $L^2 = id$.

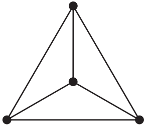
Underlying graph: $V \to$ orbits of R, $E \to$ orbits of L, faces $\to$ orbits RL on D. Incidency by intersections.

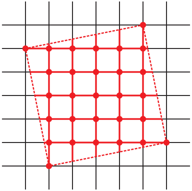
(Orientation-preserving) automorphism: $\varphi \in C_{\mathrm{Sym(D)}}(\langle R, L\rangle) = \mathrm{Aut}^+(\mathbf{M})$.

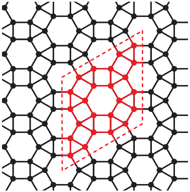
Reflections: $S = \{\varrho \ : \ \varrho \in \mathrm{Sym(D)}, R^\varrho = R^{-1}\}$.

Automorphism group of a map $\mathbf{M}$: $\mathrm{Aut}(\mathbf{M}) = \langle \mathrm{Aut}^+(\mathbf{M}), S\rangle$, $[\mathrm{Aut}(\mathbf{M}) : \mathrm{Aut}^+(\mathbf{M})] \leq 2$.

An automorphism of a (vertex-transitive) map on a surface $\mathcal{S}$ extends to a self-homeomorphism of $\mathcal{S}$;







1. Every finite group of automorphisms of a surface $\mathcal{S}$ is a group of automorphisms of a (Cayley) vertex-transitive map on $\mathcal{S}$;
2. Every finite group appears as a discrete group of automorphisms of $\mathcal{S}$ (compact, closed);
3. Not all actions of finite groups can be seen;
4. **Cyclic point stabilisers** – orientable surfaces;
5. **Dihedral point stabilisers** – non-orientable surfaces;
6. Map $\Leftrightarrow$ Action of a discrete group on an underlying surface of the map.

Main problem

Problem

Given genus g classify maps with high degree of symmetry (few orbits of $\mathrm{Aut}^+(\mathbf{M})$ or of $\mathrm{Aut}(\mathbf{M})$ on vertices, edges, faces, darts, flags,...).

Subproblem A. Classify Archimedean maps of given genus.

Subproblem B. Classify non-degenerate (polytopal, polyhedral) edge-transitive maps of given genus

Remark 1. Each highly symmetrical map defines a discrete group of automorphisms,

Remark 2. A discrete group determines such a map almost completely!!!

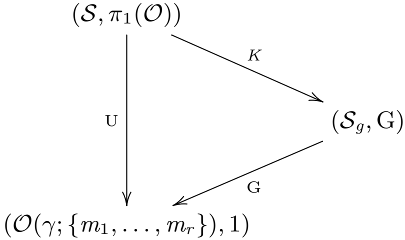
Remark 3. **Turn attention to the classification of discrete groups**, an advantage: much more applications

- Smooth coverings - classical topic in homotopy theory:
- **Regular branched coverings** between orientable surfaces, Riemann-Hurwitz equation – an action of a discrete group G is orientation preserving,

$$2-2g=|\mathrm{G}|\left(2-2\gamma-\sum_{i=1}^{r}\left(1-\frac{1}{m_i}\right)\right);\ \forall i: m_i\geq 2\in\mathbb{Z};\ m_i|\,|\mathrm{G}|\,;$$

- **Fundamental group** $\pi_1(\mathcal{S})$: elements – (eq. classes of) closed curves is $\mathcal{S}$, contractible (closed) curves are identities, operation – composition of curves;
- **Subgroup/covering correspondence**: a covering determines a subgroup of $\pi_1(\mathcal{S})$, a subgroup $\mathrm{G}\leq\pi_1(\mathcal{S})$ determines a regular covering $\widetilde{\mathcal{S}}\to\mathcal{S}$ with $CT_p\cong\mathrm{G}$,
- **Universal cover over** $\mathcal{S}$: a simply connected surface $\widetilde{\mathcal{S}}$ $(1\leq\pi_1(\mathcal{S}))$ covering all the covers of $\mathcal{S}$;
- **Fundamental group a surface of genus** g:

$$\pi_1(\mathcal{S}_g)=\langle a_1,b_1,\ldots,a_g,b_g\mid\prod_{i=1}^{g}[a_i,b_i]\rangle.$$



A discrete group G is an epimorphic image $\mathrm{G} \cong \pi_1(\mathcal{O})/K$ for a quotient orbifold $\mathcal{S}_g/\mathrm{G} = \mathcal{O}(\gamma; \{m_1, m_2, \ldots, m_r\})$.

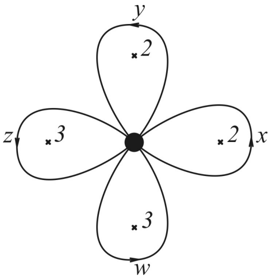
Fuchsian group $\mathrm{F} \cong \pi_1(\mathcal{O})$

$$\langle x_1, \ldots, x_r, a_1, b_1, \ldots, a_\gamma, b_\gamma \mid x_1^{m_1} = x_2^{m_2} = \ldots = x_r^{m_r} = 1, \prod_{i=1}^{\gamma}[a_i, b_i] \prod_{j=1}^{r} x_j = 1\rangle.$$

is the orbifold fundamental group $\pi_1(\mathcal{O})$.

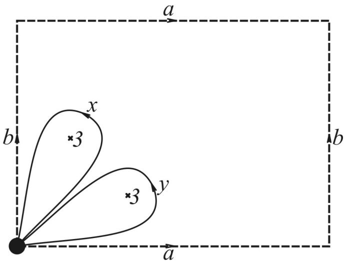
1. canonical quotient map $\bar{\mathbf{M}}$ is a bouquet of r loops on the surface of genus γ,
2. every loop is the boundary of a face containing exactly one branch-point with respective branch-index m_i,
3. outer face of the map is an $(r + 2\gamma)$-gon; contains no branch-point.

$\mathcal{O}(0; \{2, 2, 3, 3\})$



$\langle x, y, z, w \mid x^2 = y^2 = z^3 = w^3 = xyzw = 1 \rangle$

$\mathcal{O}(1; \{3, 3\})$



$\langle x, y, a, b \mid x^3 = y^3 = [a, b]xy = 1 \rangle$

Requirements: $g > 1$ - genus of a surface

Output: list of all actions of finite groups on $\mathcal{S}_g$.

(1) solve Riemann-Hurwitz equation numerically;

(2) construct Fuchsian groups $\mathrm{F}(\gamma, \{m_1, \ldots, m_r\})$ given by solutions of (1);

low-index subgroups approach

(3') search for all low-index normal subgroups of index $|G|$;

(4') for every $K \trianglelefteq \mathrm{F}$ test whether $\varepsilon : \mathrm{F} \to \mathrm{F}/K$ is order-preserving on elliptic generators of F; STOP.

or examining epimorphisms $\mathrm{F} \to \mathrm{G}$

(3") given F and G construct all epimorphisms $\mathrm{F} \to \mathrm{G}$;

(4") test whether contructed epimorphisms are order-preserving on elliptic generators of F;

(5") choose epimorphisms which will represent actions i.e. one epimorphism for a particular $K \trianglelefteq \mathrm{F}$, s.t. $\mathrm{G} \cong \mathrm{F}/K$; STOP.

Two actions determined by epimorphisms $\varphi\colon \mathrm{F} \to \mathrm{G}$ and $\psi\colon \mathrm{F} \to \mathrm{G}$ are equivalent, if there exists a group isomorphism $\alpha\colon \mathrm{G} \to \mathrm{G}$ such that for the generators x_i, a_j, b_j of particular Fuchsian group F one has

$$\begin{aligned}
\alpha(\varphi(x_i)) &= \psi(x_i), \quad i \in \{1, \dots, r\},\\
\alpha(\varphi(a_j)) &= \psi(a_j), \quad j \in \{0, \dots, \gamma\},\\
\alpha(\varphi(b_j)) &= \psi(b_j), \quad j \in \{0, \dots, \gamma\}.
\end{aligned}$$

Example

Symetric group S_3 possesses two non-equivalent actions on genus 2 surface, described by Cayley vectors

$$(y^{-1}, y, x, x)$$
$$(y^{-1}, y^{-1}, xy, x)$$

where $\mathrm{S}_3 = \langle x, y \mid x^2, y^3, (y^{-1}x)^2 \rangle$ and orbifold has signature $(0; \{3, 3, 2, 2\})$.

Rieman-Hurwitz equation

$$2-2g = |\mathrm{G}|\left(2-2\gamma-\sum_{i=1}^{r}\left(1-\frac{1}{m_i}\right)\right)$$

Criteria for a solution:

1. $\gamma \leq g$,
2. $r \leq 2g+2$,
3. $\forall i : m_i \geq 2 \in \mathbb{Z}$,
4. $\forall i : |G| \equiv 0 \mod m_i$,
5. $|G| \leq 84(g-1)$.

We obtain a set of pairs (signature of an orbifold, order of respective group)

$$|\mathrm{G}|, (\gamma; \{m_1, \ldots, m_r\}).$$

Not every signature is g-admissible: RH holds, but an action of G does not exist.

- $(0; \{7, 3, 2\})$ is not 2-admissible – no group of order 84 acts on $\mathcal{S}_2$;
- $(0; \{5, 4, 2\})$ is not 3-admissible – no group of order 20 acts on $\mathcal{S}_3$, etc. . .

$\|G\|$	Orbifold	Actions	$\|G\|$	Orbifold	Actions
1	$(2; \{\})$	1	12	$(0; \{3,2,2,2\})$	D_{12}
2	$(1; \{2,2\})$	C_2	12	$(0; \{4,4,3\})$	$C_3 : C_4$
2	$(0; \{2,2,2,2,2,2\})$	C_2	12	$(0; \{6,3,3\})$	—
3	$(1; \{3\})$	—	12	$(0; \{6,6,2\})$	$C_6 \times C_2$
3	$(0; \{3,3,3,3\})$	C_3	12	$(0; \{12,4,2\})$	—
4	$(1; \{2\})$	—	15	$(0; \{5,3,3\})$	—
4	$(0; \{2,2,2,2,2\})$	$C_2 \times C_2$	16	$(0; \{8,4,2\})$	QD_{16}
4	$(0; \{4,4,2,2\})$	C_4	18	$(0; \{18,3,2\})$	—
5	$(0; \{5,5,5\})$	C_5	20	$(0; \{5,5,2\})$	—
6	$(0; \{3,3,2,2\})$	C_6, S_3	24	$(0; \{4,3,3\})$	$SL(2,3)$
6	$(0; \{6,2,2,2\})$	—	24	$(0; \{6,4,2\})$	$(C_6 \times C_2) : C_2$
6	$(0; \{6,6,3\})$	C_6	24	$(0; \{12,3,2\})$	—
8	$(0; \{4,2,2,2\})$	D_8	30	$(0; \{10,3,2\})$	—
8	$(0; \{4,4,4\})$	Q_8	36	$(0; \{9,3,2\})$	—
8	$(0; \{8,8,2\})$	C_8	40	$(0; \{5,4,2\})$	—
9	$(0; \{9,3,3\})$	—	48	$(0; \{8,3,2\})$	$GL(2,3)$
10	$(0; \{10,5,2\})$	C_{10}	84	$(0; \{7,3,2\})$	—

Main result

Previous results

Broughton '89 classification for genera 2 and 3

Bogopolski '91 classification for genus 4

Kuribayashi and Kimura 90's classification for genus 5

State-of-art

- Abstract structure of groups and g-admissible orbifold types were dermined up to genus 21 (JK), for large groups much further (see Conder's web page, $|G| \geq 4(g-1)$)
- (almost) All actions determined by g-admissible groups are known for $2 \leq g \leq 21$ (JK);
- At present we have completed the list of actions up to genus 9, see `http://www.savbb.sk/~karabas/finacts.html`
- Small 10-admissible troublemakers with more than 8.10^5 kernels: $\mathrm{C}_2 \times \mathrm{C}_2$ of types $(1; \{2^9\})$ or $(0; \{2^{13}\})$, S_3 of types $(1; \{2^6\})$ or $(0; \{2^{10}\})$.

A sample of results: Small discrete groups

g	Signature	$\#C_2$	$\#C_4 \times C_2$	g	Signature	$\#C_2$	$\#C_4 \times C_2$
2	$(1, \{2^2\})$	4		6	$(2, \{2^6\})$	16	
2	$(0, \{2^6\})$	1		6	$(1, \{2^{10}\})$	4	
3	$(2, \{\})$	15		6	$(0, \{2^{14}\})$	1	
3	$(1, \{2^4\})$	4		7	$(4, \{\})$	255	
3	$(0, \{2^8\})$	1		7	$(3, \{2^4\})$	64	
3	$(0, \{4^2, 2^2\})$		32	7	$(2, \{2^8\})$	16	
4	$(2, \{2^2\})$	16		7	$(1, \{2^{12}\})$	4	
4	$(1, \{2^6\})$	4		7	$(0, \{2^{16}\})$	1	
4	$(0, \{2^{10}\})$	1		7	$(1, \{2^3\})$		288
5	$(3, \{\})$	63		7	$(1, \{4^2\})$		192
5	$(2, \{2^4\})$	16		7	$(0, \{4^2, 2^4\})$		320
5	$(1, \{2^8\})$	4		7	$(0, \{4^4, 2\})$		176
5	$(0, \{2^{12}\})$	1		8	$(4, \{2^2\})$	256	
5	$(1, \{2^2\})$		120	8	$(3, \{2^6\})$	64	
5	$(0, \{4^2, 2^3\})$		104	8	$(2, \{2^{10}\})$	16	
5	$(0, \{4^4\})$		48	8	$(1, \{2^{14}\})$	4	
6	$(3, \{2^2\})$	64		8	$(0, \{2^{18}\})$	1	

Discrete groups with actions on $\mathcal{S}_0$

G	$\mathrm{Epi}_{\mathcal{O}}$	Sign.	G	$\mathrm{Epi}_{\mathcal{O}}$	Sign.
1	1	$\mathcal{O}(0;\emptyset)$	A_4	24	$\mathcal{O}(0;\{2,3,3\})$
C_m	$\varphi(m)$	$\mathcal{O}(0;\{m,m\})$	S_4	24	$\mathcal{O}(0;\{2,3,4\})$
D_{2m}	$m\varphi(m)$	$\mathcal{O}(0;\{2,2,m\})$	A_5	120	$\mathcal{O}(0;\{2,3,5\})$

By RH bound, for $g > 1$ there are finitely many g-admissible orbifolds.
see `http://www.savbb.sk/~karabas/science.html#rhsu`, for $2 \leq g \leq 24$

Discrete groups with action on $\mathcal{S}_2$

G	$\mathrm{Epi}_{\mathcal{O}}$	Sign.	G	$\mathrm{Epi}_{\mathcal{O}}$	Sign.
1	1	$(2;\emptyset)$	Q_8	6	$(0;\{4,4,4\})$
C_2	1	$(0;\{2^6\})$	D_8	6	$(0;\{2^3,4\})$
C_2	4	$(1;\{2,2\})$	C_{10}	4	$(0;\{2,5,10\})$
C_3	6	$(0;\{3^4\})$	$\mathrm{C}_2 \times \mathrm{C}_6$	12	$(0;\{2,6,6\})$
C_4	2	$(0;\{2^2,4^2\})$	$\mathrm{C}_3 \rtimes \mathrm{C}_4$	2	$(0;\{3,4,4\})$
$\mathrm{C}_2 \times \mathrm{C}_2$	60	$(0;\{2^5\})$	D_{12}	6	$(0;\{2^3,3\})$
C_5	12	$(0;\{5,5,5\})$	$\mathrm{C}_8 \rtimes \mathrm{C}_2$	2	$(0;\{2,4,8\})$
C_6	2	$(0;\{3,6,6\})$	$\mathrm{C}_2 \ltimes (\mathrm{C}_2 \times \mathrm{C}_6)$	2	$(0;\{2,4,6\})$
C_6	2	$(0;\{2^2,3^2\})$	$\mathrm{SL}_2(3)$	2	$(0;\{3,3,4\})$
S_3	2	$(0;\{2^2,3^2\})$	$\mathrm{GL}_2(3)$	2	$(0;\{2,3,8\})$
C_8	4	$(0;\{2,8,8\})$			

A sample of results: Numbers of admissible pairs (group-signature)

g	# adm. pairs	max. $\|G\|$	g	# adm. pairs	max. $\|G\|$
2	21	48	14	229	1092
3	49	168	15	407	504
4	64	120	16	386	720
5	93	192	17	$>$732	1344
6	87	150	18	337	168
7	148	504	19	789	720
8	108	336	20	425	228
9	270	320	21	940	480
10	226	432	22	628	1008
11	232	240	23	716	192
12	201	120	24	625	216
13	454	360			

A sample of results: Maximal actions

g	$\|G\|$	Orbifold	G
2	48	$(0, \{8, 3, 2\})$	$\mathrm{GL}(2, 3)$
3	168	$(0, \{7, 3, 2\})$	$\mathrm{PSL}(3, 2)$
4	120	$(0, \{5, 4, 2\})$	S_5
5	192	$(0, \{8, 3, 2\})$	$(((\mathrm{C}_4 \times \mathrm{C}_2) : \mathrm{C}_4) : \mathrm{C}_3) : \mathrm{C}_2$
6	150	$(0, \{10, 3, 2\})$	$((\mathrm{C}_5 \times \mathrm{C}_5) : \mathrm{C}_3) : \mathrm{C}_2$
7	504	$(0, \{7, 3, 2\})$	$\mathrm{PSL}(2, 8)$
8	336	$(0, \{8, 3, 2\})$	$\mathrm{PSL}(3, 2) : \mathrm{C}_2$
9	320	$(0, \{5, 4, 2\})$	$(((\mathrm{C}_2 \times \mathrm{Q}_8) : \mathrm{C}_2) : \mathrm{C}_5) : \mathrm{C}_2$
10	432	$(0, \{8, 3, 2\})$	$(((\mathrm{C}_3 \times \mathrm{C}_3) : \mathrm{Q}_8) : \mathrm{C}_3) : \mathrm{C}_2$
11	240	$(0, \{6, 4, 2\})$	$\mathrm{C}_2 \times \mathrm{S}_5$
12	120	$(0, \{15, 4, 2\})$	$(\mathrm{C}_5 \times \mathrm{A}_4) : \mathrm{C}_2$
13	360	$(0, \{10, 3, 2\})$	$\mathrm{A}_5 \times \mathrm{S}_3$
14	1092	$(0, \{7, 3, 2\})$	$\mathrm{PSL}(2, 13)$
15	504	$(0, \{9, 3, 2\})$	$\mathrm{PSL}(2, 8)$

g	$\lvert G\rvert$	Orbifold	G
16	720	$(0, \{8, 3, 2\})$	$A_6 : C_2$
17	1344	$(0, \{7, 3, 2\})$	$(C_2 \times C_2 \times C_2).PSL(3, 2)$
18	168	$(0, \{21, 4, 2\})$	$(C_7 \times A_4) : C_2$
19	720	$(0, \{5, 4, 2\})$	$C_2 \times A_6$
20	228	$(0, \{6, 6, 2\})$	$C_2 \times ((C_{19} : C_3) : C_2)$
21	480	$(0, \{6, 4, 2\})$	$(C_2 \times C_2 \times A_5) : C_2$
22	1008	$(0, \{8, 3, 2\})$	$(C_3 \times PSL(3, 2)) : C_2$
23	192	$(0, \{48, 4, 2\})$	$(C_3 \times (C_{16} : C_2)) : C_2$
24	216	$(0, \{27, 4, 2\})$	$((C_2 \times C_2) : C_{27}) : C_2$

All shown maximal actions are 'triangular'

Question: Does there exist a maximal action possessing non-triangular g-admissible signature?

YES: $g = 126$, $\lvert G\rvert = 1500$, $(0; \{3, 2^3\})$ (Conder)

A map is polyhedral (Mohar and Thomassen, 2001) if its faces are bounded by simple cycles and two boundary cycles have

- empty intersection, or
- intersect in single vertex, or
- intersect in single edge.

Definition

A map $\mathbf{M}$ on a surface of genus g is **Archimedean** if it is *polyhedral* and $\mathrm{Aut}(\mathbf{M})$ is *transitive* on vertices of $\mathbf{M}$.

1. The number of vertices is bounded by $168(g-1)$ if $g>1$ (Hurwitz bound);
2. The valency is bounded by $3+\sqrt{12g-3}$ (bound is achieved by VT triangulations given by K_n);
3. (1) + (2): For every surface $\mathcal{S}_g$, $g>1$, exist finitely many Archimedean maps (on $\mathcal{S}_g$);
4. Archimedean maps up to genus 4 were classified recently (RN+JK, 2012);
5. Could we try higher genera? Probably yes...

1. Reflexibility test: $=(D; R, L)$ is reflexible iff $(D; R, L) \cong (D; R^{-1}, L)$

Theorem

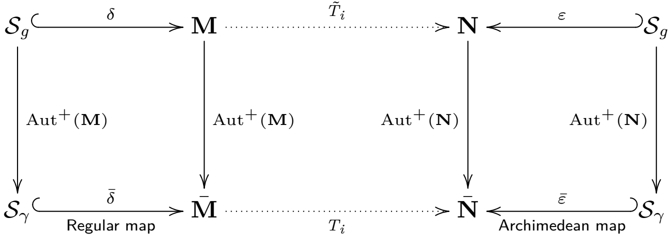
There are 13 Archimedean solids of genus two and among them
5 *are reflexible,* ***8*** *forms chiral pairs.*
There are 123 Archimedean solids of genus three and among them
39 *are reflexible,* ***84*** *forms chiral pairs.*
There are 136 Archimedean solids of genus four and among them
44 *are reflexible,* ***92*** *forms chiral pairs.*

2. Non-Cayley Archimedean solids

Theorem

There are **3** *non-Cayley Archimedean solids of genus three. There is* **unique** *non-Cayley Archimedean solid of genus four.*

3. We recognized:
   - regular maps (Conder),
   - truncations of regular maps (Conder), hypermaps (Breda and Jones)
   - medials of regular maps (Conder).



Archimedean operation defined on a (regular) map $\mathbf{M}$ is a transformation $\mathbf{M} \to \mathbf{N}$ such that

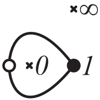
- the supporting surface of $\mathbf{M}$ and $\mathbf{N}$ is the same,
- $\mathbf{N}$ is vertex transitive;
- $\mathrm{Aut}(\mathbf{M}) \leq \mathrm{Aut}(\mathbf{N})$ and $\mathrm{Aut}(\mathbf{M})$ is vertex transitive on $\mathbf{N}$;
- $(U \circ T)(\mathbf{M}) = U(T(\mathbf{M}))$.

Theorem (Completeness Theorem, BCKN)

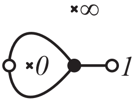
Let $\mathbf{M}$ *be a map of Archimedean class of type* $\{k, m, n\}$ *and genus* g. *Then there exist a regular hypermap* $\mathbf{H}$ *of genus* g *with* $\mathrm{Aut}^+(\mathbf{H}) = \mathrm{Aut}^+(\mathbf{M})$ *such that one of the following cases happen:*

1. $\mathbf{M} = \mathbf{H}$ *is a regular map;*
2. $\mathbf{M} \cong \mathrm{T}_j(\mathbf{H})$ *for some* $j \in \{1, 4, 6\}$, *i.e.* $\mathbf{M}$ *is medial, small snub, or quasi-snub of a regular hypermap* $\mathbf{H}$, *respectively;*
3. $n = 2$ *and* $\mathbf{M} \cong \mathrm{T}_j(\mathbf{H})$ *for some* $j \in \{2, 5\}$, *i.e.* $\mathbf{M}$ *is truncation, or snub of a regular map* $\mathbf{H}$, *respectively;*
4. $k > 3$, $m = n = 2$, $\mathbf{H}$ *is a* k*-cycle in the sphere and* $\mathbf{M} \cong \mathrm{T}_3(\mathbf{H})$ *is a* k*-antiprism;*
5. $\mathbf{H}$ *is reflexible and* $\mathbf{M} \cong \mathrm{T}_7(\mathbf{H})$ *is the flag map of a reflexible regular hypermap;*
6. $k = m$, $\mathbf{H}$ *is* $(0, \infty)^-$*-self-dual and* $\mathbf{M} \cong \mathrm{T}_j(\mathbf{H})$, *for some* $j \in \{8, 9, 10\}$, *i.e.* $\mathbf{M}$ *is the rhombic, the truncated rhombic, or the squared snub map of* $\mathbf{H}$.

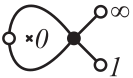
Moreover, every map of Archimedean class arises by one of the above defined costructions.



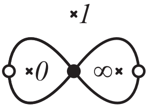
(a) Medial



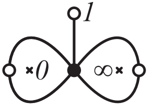
(b) Truncation



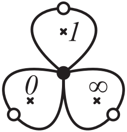
(c) Quasi-antiprism



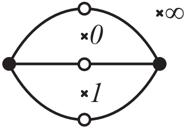
(d) Small snub



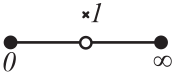
(e) Snub



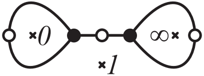
(f) Quasi-snub



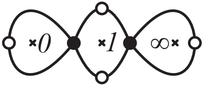
(g) Flag map



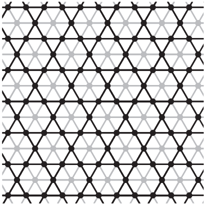
(h) Rhombic map



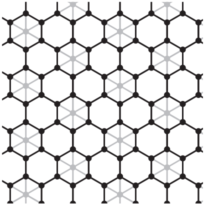
(i) Trunc. rhomb. map



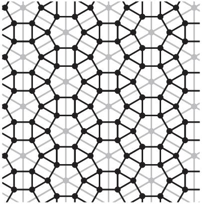
(j) Squared snub



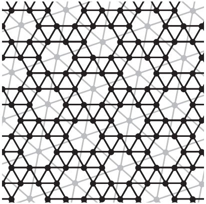
(k) Medial



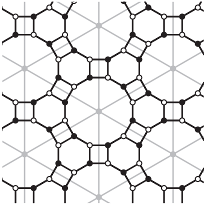
(l) Truncation



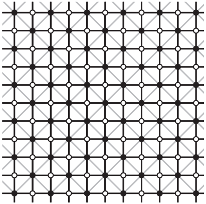
(m) Small snub



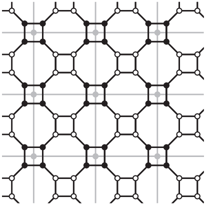
(n) Snub



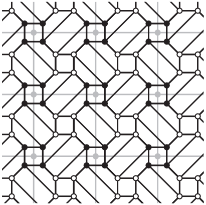
(o) Flag map



(p) Rhombic map



(q) Trunc. rhomb. map



(r) Squared snub

Problem

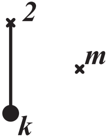
Classify edge-transitive maps of genus $g > 1$ up to isomorphism classes.

- Graver & Watkins (1997) – 14 families of maps, both orientable and non-orientable;
- Širáň, Watkins & Tucker (2001) – the families are pairwise different;
- A. Orbanić, D. Pellicer, T. Pisanski and T. Tucker (2012) – construction and classification up to $g = 4$.

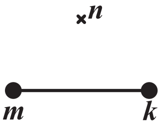
Theorem (KN)

*Let $\mathbf{M}$ be a non-degenerate edge-transitive map on an orientable surface of negative Euler characteristic. Then, up to duality, there exist **8** families of orientable non-degenerate edge-transitive maps with respect to the action of orientation preserving group of automorphisms.*

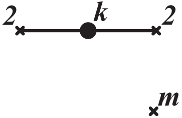
- Every family is described by its respective quotient map on a orbifold;
- 14 families are known, but theorems says something different...
- Action of Aut^+ versus action of Aut (darts vs. flags);
- Our quotients are double covers of OPPT maps: some families are merged.



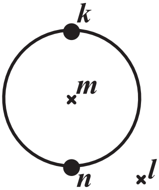
(a) E1



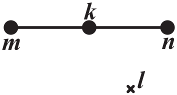
(b) E2, E2*



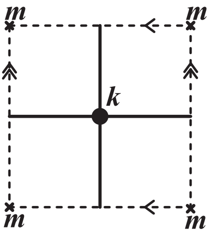
(c) E3



(d) E4



(e) E5,E5*



(f) E6